

AD-A106 964

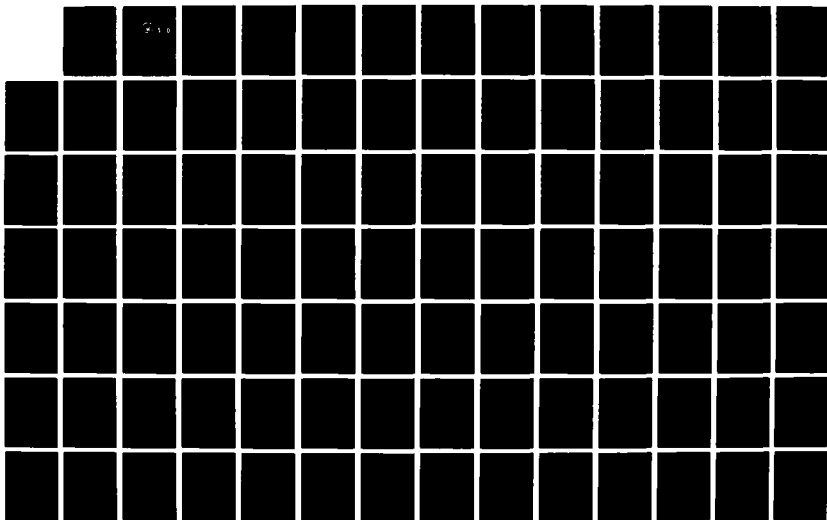
COMPARATIVE ANALYSIS OF ATTRITION GENERATION
METHODOLOGIES UTILIZED IN AGGREGATE COMBAT MODELS(U)
NAVAL POSTGRADUATE SCHOOL MONTEREY CA K L MURPHY

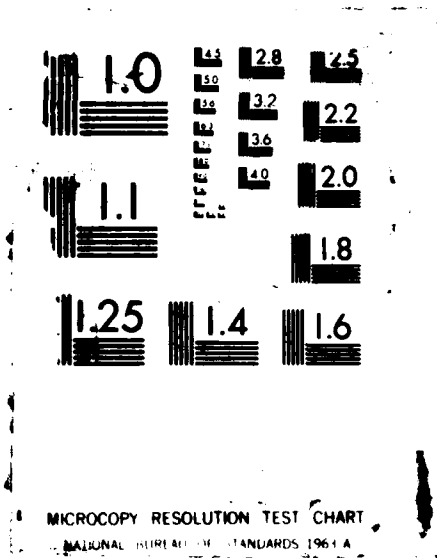
1/2

UNCLASSIFIED SEP 87

F/O 12/1

NL





AD-A186 964

2

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DTIC FILE COPY



DTIC
ELECTE
DEC 08 1987
S D
CKD

THESIS

COMPARATIVE ANALYSIS OF ATTRITION
GENERATION METHODOLOGIES UTILIZED IN
AGGREGATE COMBAT MODELS

by

Kevin L. Murphy

September 1987

Thesis Advisor: Samuel H. Parry

Approved for public release; distribution is unlimited

87 11 27 100

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
4 PERFORMING ORGANIZATION REPORT NUMBER(S)			7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School		
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School		6b OFFICE SYMBOL (if applicable) 55	7b ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000		
8a NAME OF FUNDING/SPONSORING ORGANIZATION		8b OFFICE SYMBOL (if applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000		10 SOURCE OF FUNDING NUMBERS			
		PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO
11 TITLE (Include Security Classification) COMPARATIVE ANALYSIS OF ATTRITION GENERATION METHODOLOGIES UTILIZED IN AGGREGATE COMBAT MODELS					
12 PERSONAL AUTHOR(S) MURPHY, Kevin L.					
13a TYPE OF REPORT Master's thesis		13b TIME COVERED FROM TO	14 DATE OF REPORT (Year Month Day) 1987 September		15 PAGE COUNT 129
16 SUPPLEMENTARY NOTATION					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Bonder, Subjective FPS, Analytical FPS, Attrition Coefficient		
19 ABSTRACT (Continue on reverse if necessary and identify by block number) This thesis investigates methodologies used for attrition coefficient generation in aggregate combat modes1 and the effect of these specific approaches on acceptable portrayal of combat dynamics and weapon system weighting in subsequent output interpretations. Particular attention is given to the subjective firepower scores, analytical firepower scores and the Bonder equation approaches for generation of attrition coefficients. Combat was simulated inboth Lanchester Square Law and Potential-Anti-potential aggregated models, using a common scenario for each technique. The output was compared for various individual weapons systems and overall force ratio computations. Sensitivity analysis was applied to a single system to determine reaction to various input changes and the impact on the output interpretation.					
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL PROF. SAMUEL H. PARRY			22b TELEPHONE (Include Area Code) 408-646-2779		22c OFFICE SYMBOL 55Py

Approved for public release; distribution is unlimited.

Comparative Analysis of Attrition Generation Methodologies
Utilized in Aggregate Combat Models

by

Kevin L. Murphy
Major, United States Army
B.S., United States Military Academy, 1973


Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

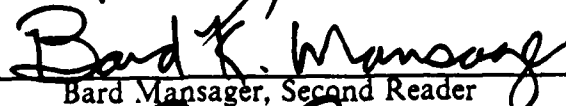
NAVAL POSTGRADUATE SCHOOL
September 1987

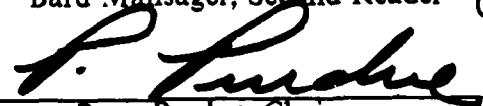
Author:

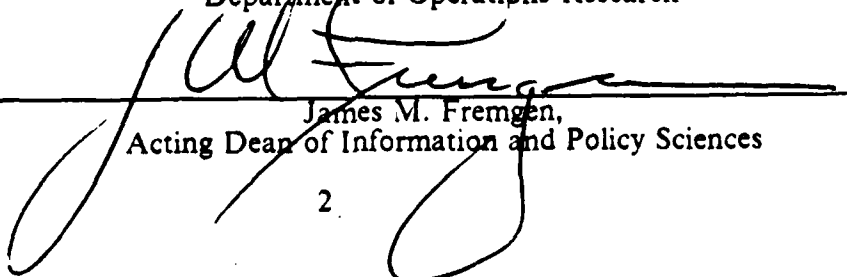

Kevin L. Murphy

Approved by:


Samuel H. Parry, Thesis Advisor


Bard Mansager, Second Reader


Peter Purdue, Chairman,
Department of Operations Research


James M. Fremgen,
Acting Dean of Information and Policy Sciences

ABSTRACT

This thesis investigates methodologies used for attrition coefficient generation in aggregate combat models and the effect of these specific approaches on acceptable portrayal of combat dynamics and weapon system weighting in subsequent output interpretations. Particular attention is given to the subjective firepower scores, analytical firepower scores and the Bonder equation approaches for generation of attrition coefficients. Combat was simulated in both Lanchester Square Law and Potential-Antipotential aggregated models, using a common scenario for each technique. The output was compared for various individual weapons systems and overall force ratio computations. Sensitivity analysis was applied to a single system to determine reaction to various input changes and the impact on the output interpretation.

Accession For	
NIS CRANI	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Date	
Approved for Release	
Date	
Limit	
A-1	

TABLE OF CONTENTS

I.	INTRODUCTION	9
A.	BACKGROUND	9
B.	AGGREGATED MODELS	10
C.	ATTRITION MODELS	10
II.	ATTRITION METHODOLOGIES	13
A.	FIREPOWER SCORES	13
1.	General	13
2.	Subjective Firepower Scoring Methods	15
3.	Analytical Firepower Scores	21
4.	Bonder Attrition Coefficients	24
5.	Combat Analysis Model (COMAN)	26
6.	Summary	28
III.	MODEL DEVELOPMENT AND EVALUATION	29
A.	OVERVIEW	29
B.	SIMULATION MODELS	29
C.	SCENARIO	31
D.	SUBJECTIVE FIREPOWER SCORES	33
E.	ANALYTICAL FIREPOWER SCORES	37
1.	Crude Analytical FPS	37
2.	Range Dependent Analytical FPS	39
F.	BONDER ATTRITION COEFFICIENTS	44
G.	SUMMARY	46
IV.	MODEL OUTPUT ANALYSIS	50
A.	OVERVIEW	50
B.	MODEL OUTPUT ANALYSIS	50
1.	Overall Force Ratios	50
2.	Individual System Analysis	52

C.	SUMMARY	71
V.	SENSITIVITY ANALYSIS	74
A.	SUBJECTIVE FPS	75
B.	ANALYTICAL FPS	77
C.	BONDER ATTRITION COEFFICIENT	80
D.	FORCE RATIO ANALYSIS	82
E.	FORCE SIZE - LARGE FORCE BATTLE SIMULATION	85
F.	SUMMARY	85
VI.	SUMMARY AND RECOMMENDATIONS	87
A.	SUBJECTIVE FIREPOWER SCORES	87
B.	ANALYTICAL FIREPOWER SCORES	88
C.	BONDER EQUATION ATTRITION COEFFICIENTS	89
D.	RECOMMENDATIONS FOR FURTHER STUDY	89
APPENDIX A:	ANALYTICAL AND SUBJECTIVE FPS SIMULATION MODEL	91
APPENDIX B:	BONDER SIMULATION MODEL	93
APPENDIX C:	ANALYTICAL FPS COMBAT RESULT	97
APPENDIX D:	ANALYTICAL FPS ATTRITION MATRICES	102
APPENDIX E:	BONDER EQUATION INPUT VALUES	109
APPENDIX F:	BONDER ATTRITION COEFFICIENT BATTLE RESULTS	118
APPENDIX G:	SENSITIVITY ANALYSIS PLOTS	119
	LIST OF REFERENCES	127
	INITIAL DISTRIBUTION LIST	128

LIST OF TABLES

1. SUBJECTIVE FIREPOWER SCORES AND INDICES	16
2. ARMY WAR COLLEGE COMBAT POWER VALUES	19
3. ANALYTICAL FIREPOWER SCORES	23
4. OPPOSING WEAPON SYSTEMS	32
5. SUBJECTIVE FIREPOWER SCORES	33
6. RELATIVE WEIGHT MATRIX	34
7. SUBJECTIVE FPS ATTRITION COEFFICIENTS	35
8. POINT OF COMBAT INEFFECTIVENESS	36
9. ANALYTICAL FIREPOWER SCORES	38
10. INDIRECT FIRE ZONE ATTRITION MATRIX	41
11. LONG-RANGE FIRE ZONE ATTRITION MATRIX	42
12. CLOSE-IN FIRE ZONE ATTRITION MATRIX	42
13. ATTRITION IN ANALYTICAL FPS MODELS	43
14. BONDER EQUATION GENERATED ATTRITION MATRICES	47
15. ATTRITION IN BONDER COEFFICIENT MODEL	47
16. IMPROVED 25MM GUN SYSTEM	75
17. REGRESSION ANALYSIS OF WEAPON SYSTEM TRADE-OFFS	76
18. RELATIVE WORTH MATRICES	97
19. INITIAL ATTRITION COEFFICIENTS	98
20. SIMULATION RESULTS FOR INITIAL MATRICES	98
21. SIMULATION RESULTS FOR 0.01 REDUCED MATRICES	99
22. SIMULATION RESULTS FOR .005 REDUCTION	100
23. SIMULATION RESULTS FOR .0025 REDUCTION	101
24. A_{IJ}	103
25. B_{IJ}	105
26. ANALYTICAL FPS SIMULATION RESULTS	107
27. RED FORCE INPUT VECTORS	109
28. CONDITIONAL PROBABILITY TABLES FOR FOLLOW-ON HITS	110

29. CONDITIONAL KILL PROBABILITIES FOR RED FORCE SYSTEMS	112
30. BLUE FORCE INPUT VECTORS	113
31. CONDITIONAL PROBABILITY TABLES FOR FOLLOW-ON HITS	114
32. CONDITIONAL KILL PROBABILITIES FOR BLUE FORCE SYSTEMS	116
33. BOMBER ATTRITION COEFFICIENT SIMULATION RESULTS	118

LIST OF FIGURES

4.1	Force Ratios	51
4.2	Lanchester Model Combat Flow - M1 Tank	53
4.3	Lanchester Model - ITV Combat Flow	55
4.4	Potential-Antipotential Model - ITV System Values	56
4.5	Lanchester Model - M2 IFV Combat Flow	57
4.6	Potential-Antipotential Model - M2 IFV System Values	59
4.7	Lanchester Model - M109A2 Howitzer Combat Flow	60
4.8	Potential-Antipotential Model - M109A2 Howitzer	61
4.9	Lanchester Model - T72 Tank Combat Flow	64
4.10	Potential-Antipotential Model - T72 System Value	65
4.11	Potential-Antipotential Model - AT5 ATGM System Values	68
4.12	Lanchester Model - BMP IFV Combat Flow	69
4.13	Potential-Antipotential Model - BMP System Value	70
5.1	Comparison of M2 Systems In the P-AP Model	79
5.2	Comparison of Force Ratios - Analytical FPS Method	83
5.3	Force Ratio Changes in Bonder P-AP Model	84

I. INTRODUCTION

A. BACKGROUND

In November 1986, the Deputy Under Secretary of the Army for Operations Research (DUSA-OR) sponsored the Combat Scoring System Workshop. The objective of the workshop was to "discuss and evaluate existing weapon scoring systems" and estimate characteristics for future modelling systems. One conclusion drawn from the three day meeting was that the current static, linear aggregated scoring systems will be used until more acceptable alternatives can be produced. During the workshop, discussions were divided into the areas of the generation and interpretation aspects of combat models. [Ref. 1: p. 28]

The generation component of any model produces quantitative combat measures of effectiveness. Within this component, the attrition process possesses differing degrees of importance relative to the scope of the model. The importance of the attrition process generally decreases as the force size increases and the non-attrition processes such as C³I, logistic sustainability, and mobilization become the major command issues. However, the cornerstone of a generation model is its ability to represent the dynamics of combat, of which the attrition process is a principal driving force. While the actual importance of the attrition process within the model may decrease as the model resolution levels increase from battalion to theatre level, they still provide input that supports the decision process at division, corps and theatre. Hence, products from the generation model such as killer/victim scoreboards serve as key inputs to the interpretation models. As such, the models used to portray the combat attrition process must be based on acceptable generation methodologies which reflect the dynamics of combat over time. This same characteristic must be inherent within the attrition processes of these aggregated models if believability in the models is to be maintained.

This thesis will investigate the aspects of various attrition generation methodologies in light of their ability to portray the dynamics of combat over time and the effect of their output on the interpretation process.

B. AGGREGATED MODELS

There are two basic categories or classifications for aggregated models: homogeneous and heterogeneous. Homogeneous aggregation combines the combat power of the unit's components into a single measure of combat power. Computations are then based on the relative combat power of the opposing forces using a ratio of their combat power indices. In heterogeneous aggregated models, the effectiveness of a firer weapon type against different opposing force systems/targets is modelled. The unit entities then maintain a count of surviving systems for each time step until a combat resolution or decision criteria has been met and combat is discontinued. Current trends in models favor the heterogeneous approach as it allows more accurate attrition modelling than homogeneous aggregation. Further, the extensive averaging required to develop a single unit's combat power index results in increased information loss, which is the major disadvantage in using aggregated models [Ref. 2: p.1-8]. Likewise, it is important to note, as Taylor pointed out, that the heterogeneous approach used in Lanchester type models is just an extension of the homogeneous, and what is used in one may be extended for use in the other [Ref. 3: p.87]. Consequently, the same methodologies employed in homogeneous aggregation can be utilized to determine attrition factors in heterogeneous aggregation models.

C. ATTRITION MODELS

Attrition models can be categorized as being static or dynamic, as well heterogeneous or homogeneous. Homogeneous and heterogeneous classifications adhere to the same tenets used in defining the nature of aggregation models. Static scoring models compute a unit's combat worth through force comparisons derived from firepower scores, weapon effectiveness indicators/weighted unit values (WEI/WUV), and force ratios. In such a static analysis, forces are defined, weapon scores are determined, aggregated index values summed and a force ratio computed. The resulting comparison is considered to be representative of the combat capabilities of the opposing forces. Such an approach to attrition modelling is considered attractive because of its simplicity to execute and interpret. The static method negates the dynamics of combat through successive averaging of factors for weapon systems and units into the basic situational tactics such as attack, defend, and delay. The dynamic scoring approach determines the value of a unit or system at each step of the simulation based on various definable parameters such as range, time to target acquisition, and technical or engineering data. The Bonder equation for determination

of attrition rates is an example where the dynamics of the battlefield environment and technical data are incorporated into the attrition process. This is accomplished by using data from the subroutines in the force-on-force simulation model. Through the various submodules, the situation for deciding which data values to be used in the attrition process are determined. The detailed subroutines for movement, resupply and the other combat attributes that are modeled provide the conditions which are used to assess the values for the variables used in the Bonder equation. Therefore, while the Bonder equation is only a means to assess attrition, its interaction with the simulation model subroutines results in the use of dynamic rather than static input values. -

The majority of large scale, aggregated models utilize attrition methods which are linear. Within these models, most approaches assign values to weapon systems based on some variation of what can be generically called firepower scores or the Potential-Antipotential approach. Several models have emerged that make use of less conventional approaches, such as the Attrition Calibration Model (ATCAL), which incorporates a nonlinear approach to some of the combat processes [Ref. 2: p.6-13] ; however, it is a derivation of the Potential-Antipotential approach. The Combat Analysis Model (COMAN), developed by Clark in 1969, and its successor the Combat Analysis Model Extended (COMANEX), utilizes subsystem performance factors of the systems from high resolution simulation battles and internally aggregates the results to achieve an attrition coefficient/firepower score [Ref. 4: p.54-58]. A third methodology in use today, developed by Bonder, creates the attrition rates by using externally precomputed performance parameters to yield the time to defeat a target for various systems. In both cases, the times between casualties and target defeat are used to develop specific attrition coefficients. Specific attention will be given to the subjective firepower score approach, the analytical firepower score approach, and the Bonder approach.

While the basic categories for attrition models have been introduced above, the basic concept of the attrition process needs to be defined before discussion and analysis of specific methodologies is undertaken. Simply stated, an attrition process is defined as the means by which values are imputed to a weapon system/unit and incorporated into a simulation model to determine the outcome of a simulated battle. The attrition method may use a series of simple or complex equations to represent combat between weapon systems or units. A simple but concise mathematical expression for the attrition process is

$$Y \text{ casualties} = (X \text{ firers}) \times (\text{Attrition Rate}) \times (\text{Time})$$

While this equation can be considered dimensionally correct, the level of realism or degree of combat dynamics portrayed in the model is concealed within the attrition rate coefficient. Consequently, the method use for calculating the attrition coefficient becomes more important as it simulates or governs the actions of individual and groups of combatants below the interpretive resolution level of the model.

Chapter II examines the firepower score, Bonder and COMAN approaches for computing relative combat values for weapons and the associated attrition coefficients. Chapter III outlines the simulation procedures and evaluates the various methodologies when used in a Lanchester Square Law and Potential-Antipōtential simulation models. Chapter IV examines the results of simulation tests and Chapter V investigates the sensitivity of the techniques to change and their impact on the decision maker. Chapter VI summarizes salient points observed for the respective techniques and their use in future aggregate models.

II. ATTRITION METHODOLOGIES

A. FIREPOWER SCORES

1. General

Lester and Robinson [Ref. 5: p.4] define a firepower score (FPS) as the relative value of a weapon based on its firepower. The firepower index (FPI) for a unit is achieved by summing the firepower scores of the weapons within the unit of resolution. Thus, the firepower index of a unit is a linear sum of the firepower scores and represents the aggregation of all weapon systems within the force. The extent of the aggregation, as previously noted, may result in a single overall force value in the case of a homogeneous model or several values by weapon system types or unit type in heterogeneous models. Regardless of which aggregation is utilized, the general linear formula for a firepower index is

$$FPI = \sum X_i S_i \quad (\text{eqn 2.1})$$

where X_i = the number of weapon type i in the force

S_i = the FPS (combat value) of weapon type i [Ref. 2: p. 4-6].

It should be noted at this point that even though firepower scores and firepower indices are related, they are not synonymous. Firepower scores apply to the weapon systems and the indices to units. Further, while there exists a general equation for calculating firepower indices, attempts to develop acceptable techniques for the computation of firepower scores/values have spawned numerous approaches, but none which have been able to capture the complexities of combat.

As noted above, the approaches used to determine numeric values for firepower scores have never fully reflected the dynamics of combat, yet they continue to be widely used in models as attrition rate coefficients. The approaches for determining firepower scores and indices developed over the years can be placed into six general categories [Ref. 2: p.4-8] :

- Measures of perceived combat value
- Measures of combat performance
- Measures of multiple characteristics of the weapon system
- Measures of weapon lethality

- Measures of mission dependent lethality
- Measures of weapon kill potential.

For purposes of this paper, these six categories will be reduced to three classes; those using subjective evaluations as the primary analytical tool, those using lethality, and those using weapon potential for assignment of firepower scores or attrition coefficients.

The categories which are considered as subjective are those using measures of perceived combat value, measures of historical combat performance, and multiple characteristics of weapon systems. Measures of perceived combat value, grouped into the Subjective FPS category, are derived from military judgements and experience. Such judgements correspond to those used in the planning and assessment process of military operations. An example of such an approach is assigning a value to a U.S. mechanized infantry division that is 1.5 times greater than the value of a British infantry division. Measures of historical combat performance use casualty figures attributed to specific weapon types from WWII and Korea to assign values to current weapons. How this data is transformed to account for changes in weapon lethality, development of new weapon systems and situational tactics is a subject in itself and will not be pursued in this paper. The multiple characteristic approach, developed in the late 1960s and early 1970s, combined numerous weapon characteristics such as mobility and survivability with the firepower/lethality of the system. Factors are combined through a linear weighting technique using Delphi analysis. The WEI/WUV scoring system is such an approach.

Analytical firepower scoring methodologies are comprised of measures of weapon lethality and mission dependent weapon lethality. Measures of weapon lethality assign values based on the relative killing power of the weapons. Values are developed from ammunition expenditure rates and lethal area/kill probabilities. Modification of these values based on the posture of the force i.e. mission, terrain, etc. are used to bring the firepower scores closer to the realities of the combat environment. Mission dependent weapon lethality applies modifying factors before calculation of values, hence judgemental factors are incorporated to take into consideration major situational conditions such as offensive and defensive postures and target acquisition.

The final category, measures of weapon kill potential, determines the value for a weapon by what it can kill on the battlefield. Specifically, it defines the firepower score of a weapon as being proportional to the sum of the scores of all enemy systems

it kills. This leads to an interactive system of eigenvalue equations which are solved at each time step for the new weapon values. These values are highly situation dependent and are evaluated in the context of specific scenarios. This computational procedure is called the Potential-Antipotential Method. The development of values used for this method are derived from any number of approaches, some of which are listed above.

Some of the methods listed above are to varying degrees dynamic. As noted in Chapter 1, attrition models may be either static or dynamic. Firepower scoring, the assignment of a single value for a unit, results in static force comparisons. The force ratios derived from the firepower scores and firepower indices are inputs for force comparisons. Depending on the specific firepower scoring methodology used, firepower scores generally provide a simplified estimate of large unit combat capabilities and not weapon system interaction. Static models remain attractive to the casual user because of the simplicity of computations and interpretation, but are of questionable value for providing answers to more specific questions.

2. Subjective Firepower Scoring Methods

a. General

Subjective firepower scoring has been widely used in early simulation models as a means to develop Lanchester-type attrition coefficients. The approach uses a committee-type structure, sometimes referred to as a Delphi technique, to assign firepower scores/values to weapon systems over a given scale. The value assigned encompasses the entire range of activities and capabilities and is fixed throughout any subsequent phase or evaluation process. The firepower values could be assigned for homogeneous organizations or for individual weapon types in heterogeneous models. However, it should be obvious that the relative worth of weapons and units in various stages of battle, i.e. indirect fires, long range fires, and and close-in fire zones, are not adequately portrayed by single assessments. Further, composition of the assessment committee could bias the assigned values as well as preclude consistent and acceptable replication by different committees or study groups. This creates a fundamental weakness when using the basic subjective firepower score methodology in any model. More detailed analysis reveals that specific effects for factors such as terrain, posture (attack, defend, etc) and force mix are not always considered. More succinctly stated, subjective firepower scores are developed from the summation and products of numerous combat modifiers which represent the perceived contribution of an activity to the unit's worth over the entire spectrum and duration of combat. Consequently,

variations on the basic subjective scoring approach were developed over the years in an attempt to capture the flavor of battle based on the incorporation of more attributes into the force value computations. Methodologies encompassed by the subjective approach include WEI/WUVs, the Army War College Combat Power Scores (AWC), and the Quantified Judgement Methods (QJM).

b. Subjective Firepower Scores and Indices

The most basic form of the subjective firepower score approach is a straightforward assignment of perceived values to weapon systems. These values are bounded over an arbitrarily selected range and the units scored in accordance with the general FPI equation (Eqn 2.1). The final ratio of firepower indices, the force ratio, developed by the Subjective technique is then utilized as a surrogate for the attrition coefficient, a or b , in a Lanchester-type simulation. Thus, the surrogate attrition coefficient generated by a subjective firepower approach is defined as

$$a = \frac{\text{FPI(A)}}{\text{FPI(B)}} \quad (\text{eqn 2.2})$$

An example of the firepower scores and indices for a battalion size task force developed by this method are shown in Table 1.

TABLE 1
SUBJECTIVE FIREPOWER SCORES AND INDICES

WEAPON TYPE	VALUE(S_i)	QUANTITY(X_i)	$S_i X_i$
M1 Tank	100	42	4200
ITV	50	10	500
M2 IFV	60	6	360
155 How	15	6	90
203 How	15	4	60
INDEX =			5210

c. Weapons Effectiveness Indices/Weighted Unit Value (WEI/WUV)

The WEI methodology divides weapons into seven basic categories, called families, ranging from small arms to artillery. A set of dominant characteristics is

defined for each family, with the number of characteristics varying among each family. Characteristics, such as firepower, mobility, reliability and survivability, previously unquantified in the basic methodology, are incorporated as weighted variables which when summed establish a WEI. The general expression for the WEI is [Ref. 5: p.12]:

$$WEI = W_1 C_1 + W_2 C_2 + \dots + W_n C_n$$

where W_i = subjective weight of weapon family i

C_i = dominant characteristic

$\sum W_i = 1.0$

n = number of weapon families

Each dominant characteristic, C_i , is likewise factored into distinct components, S_c , relating to such factors as lethality, ammunition availability, auxiliary weapons, night fighting capabilities, etc. These factors are quantified on a basis of 0 - 1.0 for all the weapons in the given family and a ratio for each weapon against a selected standard family weapon is computed. The equation for dominant characteristics is: [Ref. 6: p.38]

$$C_i = \sum d_i S_c$$

where d_i = weighted dominance ($\sum d_i = 1.0$)

S_c = normalized subcharacteristic of weapon i

Weighted Effectiveness Indices originally were measures of relative values within weapon groupings, and not representative of relative combat capabilities. In order to use WEI values in the broader heterogeneous scoring approach, each family of weapon was assigned a relative weight. The product of the associated family weight and the WEI for each weapon in the family yields a combat worth factor. Summing of combat worth values in turn yields the weighted unit value (WUV), which corresponds to the FPI used in the basic Subjective FPS methodology.

While it appears that the WEI/WUV approach quantifies the characteristics of the battlefield environment, it is important to remember that the basis for most of the principal aspects of the methodology are dominated by judgement and not engineering technical data. Closer examination of WEI/WUVs reveals that, like other subjective firepower scores, they are constants, fixed for the duration of combat and not time related functions of the combat posture of the force. Further, while attempting to quantify various aspects of combat, WEI/WUVs neither reflect the effects of terrain nor include all the weapon systems encountered on the battlefield, e.g. mines. The result of this more detailed approach is a set of values which still employ

generalized factors in an attempt to model the dynamic facets of the battlefield. It is important to note that while the WEI/WUV technique prescribes a more disciplined and analytical approach in determining the FPS and FPIs, the end product that is used to describe an attrition coefficient is a ratio of aggregated, averaged factors designed to measure relative value and not the capability to destroy an opposing target or system.

d. Army War College Combat Power Values (AWC)

Created as input for the Research Analysis Corporation's Theatre Combat Model/CONAF Evaluation Model, students from the Army War College developed a series of judgemental combat power scores for U.S versus Soviet combat units. The scores represent the relative value of armor, artillery, and infantry units in seven different mission postures. The methodology assigns a base unit (in this case a U.S. armor battalion with mission type i and terrain type j) a single arbitrary combat value. All other units values are derived from this base unit through the use of a Delphi technique.

The selected base unit represents the force that would have the optimal combat power value under favorable mission and terrain conditions. It is then evaluated against the remaining six mission postures, within the attack, defend, or meeting engagement categories. Other U.S. units are then subjectively evaluated for their ability to perform similar missions under the circumstances inherent in the mission posture. Effectiveness of units to perform each mission considers such factors as ability to provide long range fires, utilize cover and concealment, vulnerability to opposing forces, time to organize, and contribution to overall combat power of the force. External modifiers such as terrain and force mix effects are based on the further subjective assessments of their effect they have on unit performing one of the specified combat missions (e.g. mountainous terrain may have a 0.3 factor for armor units but only a 0.9 for dismounted infantry). The AWC scores are then multiplied by these modifiers to produce the combat scores of the unit for a given scenario. Opposing force values are determined through a comparison of Tables of Organization and Equipment (TO&E) from which the ratio of key equipment became the weighting factor (e.g. a Soviet tank regiment with 94 tanks would have a combat value approximately 1.75 times greater than a U.S tank battalion with 54 tanks). Task forces and larger unit values are the summation of the values assigned to their subordinates unit. Consequently, a series of relative combat scores for the various combat missions of forces in combat are tabulated. A sample of the unit values developed using the AWC approach is presented in Table 2. [Ref. 5: p.21]

TABLE 2
ARMY WAR COLLEGE COMBAT POWER VALUES

	DEFEND			ATTACK			Meeting
	OrgDef	Delay	HastyDef	OrgDef	Delay	HastyDef	Engagement
U.S.							
MechBn	18	7	14	10	6	12	6
TankBn	30	20	28	22	16	24	16
ArtyBn	12	8	10	8	6	9	6
Soviet							
MRR	35	18.4	29.5	22.7	15.2	25.6	17.2
TkRegt	53	35	49	38.7	28.1	42	28.1
122Bn	12	8	10	8	6	9	6

These values are then transformed by terrain and force organization factors within the larger model to achieve relative force values based on the additional factors of mission and terrain not incorporated in WEI/WUV.

While the approach allows the nonlinear aspects of interaction between forces, mission type and terrain to be represented, the method directs its attention to unit and not weapon system interaction. This approach leads to several shortfalls in the interpretation of model outputs. First, the dependence on judgemental factors throughout the table development process precludes adjustment of unit scores for weapon mix or effectiveness without a costly investment of time. This is further complicated by the fact that the implicit judgements used to develop the original scores are almost impossible to verify, so proper/correct consideration of changes in force structure and weapon capabilities are difficult and open to questions of validity. The end result is that little or no information is developed about the relative value of weapons. A second shortfall is that the aggregation process is ill-defined and, as such, limits force comparison information to the decision maker. The proportional approach to U.S. and Soviet units fails to take into account the differences in weapon capabilities. Although appearing to encompass most of the key aspects of the modern battlefield, the method still renders a combat value that only varies with a mission and terrain condition. Simply stated, the AWC approach replaces the single combat value of previous subjective methodologies with several optional values subject to the level of terrain modelling and mission assignment algorithms, breaking the battle into several

phases with static values rather a single number for the entire simulation process. Like previous techniques, the end products are unit oriented and present situational force ratios based on weighted values and not weapon system interactions.

e. Quantified Judgment Method (QJM)

The QJM falls under the measures of combat performance category for firepower score generation. It is a general method of interpreting historical data and predicting the relative performance of current and future forces. This is accomplished through the development of a set of theoretical weapon lethality indices (TLI) which define the potential capability scores for various weapons in a force. Weapon and force modifiers, developed through a series of equations, are applied to bring simulation outcomes, using the TLIs, into agreement with outcomes from a set of historical battles. Having 'tuned' the equations and modifiers, introduction of new equipment characteristics into the model should produce the outcome of a hypothetical battle. Hence, the effect of new weapons can be analyzed for a general battle scenario.

The TLI equations are developed for various weapon groupings such as crew-served and mobile weapons. The TLI for a crew served weapon is

$$\begin{aligned} \text{TLI} = & (\text{sustained rate of fire}) \times (\text{target density}) \times (\text{kill probability}) \\ & \times (\text{effective range}) \times (\text{accuracy}) \times (\text{reliability}) \end{aligned}$$

with a base target density of 1 man per 4 sq ft derived from the characteristics of the pre-Roman phalanx. The TLIs are then modified by applying a dispersion factor which relates the expected modern battlefield densities in nuclear or nonnuclear scenarios. The TLIs are divided by the dispersion factor to yield an Operational Lethality Index (OLI). The weapon effectiveness of a force is the sum of these OLIs multiplied by terrain, weather, season, and air superiority factors wherever they exert significant influence upon operational lethality. Consequently, infantry weapons may be modified by a terrain factor, while armor weapons are affected by terrain, weather, season and air superiority. The result is a force strength value. [Ref. 5: p.24]

Strength values are modified by operational variables such as mobility, training status, mission posture, and vulnerability. The resulting value is the combat potential of the force. The complexity of the operational variable equations varies from simple formulas using constants and technical data to extensive and complicated equations. Unfortunately, the level of subjectivity is markedly high throughout the approach and use of regression techniques and other statistical analysis methods is largely ignored by T. DuPuy, the developer of this methodology.

The ratio of combat potentials of opposing forces is then used to determine the winner of the battle. Ratios greater than 1.0 indicate the friendly force should achieve its objectives, while a value less than 1.0 would indicate mission failure for the friendly force. The single value force ratio which is output from the model provides little insight for weapon or force comparison decision requirements. It indicates that a new weapon or system will be more or less effective than its predecessor but the degree of effectiveness cannot be determined without utilizing some external scaling methodology. The values for TLIs, OLIs, combat potential, etc are developed through the use of modifiers that relate the systems to a set of historical battles. The modifiers do not result in exact fits to the historical battle but to some unspecified degree of closeness. The magnitude of the closeness of fit will naturally bias the final output. Further, the estimation process for determining new weapon characteristics is not specified and, like the major portion of the approach, highly subjective. While the approach attempts to account for the gamut of combat interactions, its end product is a firepower score that is based on averages and not the individual attrition dynamics of the battlefield.

f. Subjective Firepower Score Summary

Regardless of which subjective technique is used, the resulting weighted value for a force or weapon system represents an overview of the entire combat environment. These approaches, through the use of judgemental evaluation and modification of scores, are averaging the various aspects of combat over the entire battlefield and then further aggregating these to achieve a firepower index. This multiple folding of averaged quantities, each of which is based on an 'averaged' judgement, neglects the time dependent value of weapons and subordinate units. The force ratio and firepower scores do not model the attrition process but represent a static, dimensionless measure of effectiveness for the system or unit across a battle or for one set of situations. Therefore, the use of a measure of effectiveness as an input to the attrition process is logically unsound.

3. Analytical Firepower Scores

a. Overview

Concurrent with the evolution of the subjective approaches for determining a weapon's value, a more analytical approach was developed in connection with the ATLAS model. The approach uses the firepower potential of a weapon as a measure of its value. The methodology determines weapon scores as the product of expected

ammunition expenditure and the lethal area per round fired. [Ref. 5: p.5] Scores for units are the sum of the firepower potential scores for the individual weapons in the unit. The data used for the expected expenditure rates and lethal area are extracted from field manuals, technical reports or derived from historical data. While some of the data input may be challenged in terms of currency, modification of data for rates of fire (ammunition expenditure) and lethality is relatively uncomplicated when compared to the techniques used in the subjective FPS methodologies. One major area of uncertainty with this type of approach is portrayal of the relationship between the lethality-of area, point fire, and guided munition (antitank weapons) to a unit's score. A second area of noted weakness is that the synergistic effects between weapon classes are essentially neglected.

The analytical firepower score for an area fire weapon (artillery) is defined as

$$S_i = (\text{daily ammunition expenditure}) \times (\text{lethal area per round})$$

and for point fire weapons and guided munitions as

$$S_i = (\text{daily ammunition expenditure}) \times (\text{probability of single shot kill})$$

Although the values for ammunition expenditure rates are based on empirical data and extracted from published planning tables, they remain situationally dependent and adjustments to the rate of fire during the the conduct of a battle rely on judgmental and not doctrinal factors. Introduction of judgemental factors lessen the methodology's ability to accounting for some aspects of combat dynamics. [Ref. 2: p.4-9]

Computation of firepower scores using analytical and engineering data from the Ballistic Research Laboratory(BRL) and Army Materiel Systems Analysis Activity (AMSAA) provide up-to-date system characteristics of modern weapons, and help limit the uncertainty to definable limits for such values as range dependent probability of kills. This contributes to increasing the level of believability, which is always a goal in simulation models, for models using this method. Use of doctrinal publications such as FM 101-10-1 provides the necessary base to determine mission related expenditure rates and allow integration of situational considerations into the generation process rather than as an external modifier. The product of these data sets is a situationally relevant firepower score based on actual or projected weapon characteristics and lethality potential. Summation of these individual scores provides the same type scores for units, allowing model output to be analyzed for both weapon

and unit effects. An example of individual firepower scores generated by this technique for various weapon system is shown in Table 3. The firepower index, or unit firepower potential score, is achieved by summing the product of the weapon firepower score and the quantity of weapons within the organization.

TABLE 3
ANALYTICAL FIREPOWER SCORES

System	P(kill)	Ammo Expend./Wpn	S_i
M1 Tank	.35	40	14.0
M2 IFV	.55	6	3.3
M109A2	.03	60	1.8
T72 Tank	.70	40	28.0
BMP-1	.40	25	10.0
AT5 ATGM	.80	10	8.0

P(kill) are based on an attacking US force
and defending Soviet force

b. Advantages and Disadvantages

While the analytical FPS method more clearly defines the processes and interrelationships that comprise the nature of combat than the subjective FPS technique, both methods produce a single dimensionless number that is used as a substitute for an attrition rate which measures casualties per unit of time. Another weakness in both approaches is their assumption of linearity. Increasing the number of weapons or ammunition expenditure rates will result in a proportional increase in casualties which is not consistent with actual combat experiences. Further, evidence indicates that such changes have a significant influence on the relative value of the weapons. Finally, it is obvious from the formulation that the process does not incorporate all the effects that factors such as movement, mission posture, and supply status have on the attrition rate. [Ref. 5: p.10]

If the analytical FPS technique fails to incorporate a large portion of the factors inherent in the composition of a units combat potential, it, in turn, does provide a framework that allows a more analytical and systematic approach for inclusion of these factors into the aggregation process. Models such as ATLAS relate the effect of attrition on these factors and the influence the factors have on the attrition rate. Additionally, the use of a clearly defined base for firepower results in a measure of relative value or weight for a force or unit. This distinction enhances

analysis by providing a relative scale of measurement by which units are compared. Derivation of attrition coefficients from firepower scores using analytical techniques more closely represent the actual attrition process than values derived from subjective force ratios, as destructive potential or capacity of a unit or system is less likely to be neglected by other modelling factors

c. Subjective and Analytical Firepower Score Conclusion

Both firepower scoring approaches attempt to generate attrition coefficients based on the Lanchester definition that 'casualty rates are directly proportional to the number of firers' [Ref. 3: p.8]. However, regardless of how the casualty attrition process is developed within the FPS technique, the outputs are evaluations of relative value, not rates, and should not be used as attrition coefficients in aggregated models.

4. Bonder Attrition Coefficients

The basic concept for casualty assessment attributed to Lanchester-type Square Law models is that the casualty rate is directly proportional to the number of firers in the opposing forces. The Lanchester attrition coefficient, a , denotes the rate at which a typical X firer kills a Y firer over a period of time, t . This coefficient may be a function of time, force size, or any number of scaling factors depending on the assumptions used to model the process. In the Bonder approach, the attrition rate coefficient is expressed as the reciprocal of the expected time between casualties;

$$a = \frac{1}{E[T_{XY}]} \quad (\text{eqn 2.3})$$

where T_{XY} = the time for a Y firer to kill an X target

Extension of the single firer, homogeneous viewpoint expressed above follows the same aggregation methodology used in previous techniques. Therefore, the casualty rate is defined as the 'product of the single-weapon-system-type kill rate and the number of enemy firers' [Ref. 3: p.10]. At this point, the Bonder method clearly diverges from the previous techniques. Whereas the subjective and analytical FPS approaches assumed a constant kill rate over a period of time, Bonder ties the attrition coefficient to time dependent factors which are adjusted to fit the flow of battle. Most importantly, Bonder looks at only the attrition process and avoids modifying the resulting values with peripheral adjustments based on subjective assessments.

The basic equation developed by Bonder to determine the expected time to kill a target $E[T]$ can be computed by either summing the component event times leading to a combat kill or based on a first passage time semi-Markov process. Taylor has shown [Ref. 3: p.25-29] regardless of the method used within the simulation model for the generation of the attrition coefficient, either approach yields the expected time to kill equation:

$$E[T] = t_a + t_1 + t_h + A1 + A2[A3 + p(h|h) - p_1] \quad (\text{eqn 2.4})$$

$$A1 = \frac{(t_h + t_f)}{p(k|h)}$$

$$A2 = \frac{(t_m + t_f)}{p(h|m)}$$

$$A3 = \frac{(1 - p(h|h))}{p(k|h)}$$

where

- t_a = time for target acquisition
- t_1 = time to fire the first round at each new target
- t_f = time of projectile flight to target
- t_h = time to fire a round after a hit
- t_m = time to fire a round after a miss
- $p(k|h)$ = probability of a kill given a hit
- $p(h|h)$ = probability of a hit given a hit on the previous round
- $p(h|m)$ = probability of a hit given a miss on the previous round
- p_1 = probability of a first round hit

The equation can be used for deterministic quantities and probabilistic variables when expected values are substituted. In this manner, the degree of resolution for any variable is governed by the detail of the generating algorithm and not the number of modifying factors. Regardless of the complexity of the process, a 'time to kill' is produced and an attrition coefficient calculated. Operational factors such as target acquisition, range-dependent weapon-system capabilities, unit decision points, and line of sight (terrain) are developed through various techniques and exact procedures. The modelling of these factors is not of primal importance in the analysis of the attrition coefficient generation process. The significance of how these aspects of combat are modelled (e.g. serial or parallel target acquisition or burst versus volley

fire) is that different techniques will generate different attrition coefficients, not because the expected time to kill equation varies, but simply because an input variable is changed.

The major advantage of generating attrition coefficients using this approach is that the coefficients are computed from measurable weapon characteristics and have a high degree of validity. Additionally, this technique is explicit, provides an easily interpreted audit trail, and does not rely on judgemental tuning factors or external modifiers so heavily relied upon by the firepower approaches [Ref. 3: p.83]. An additional consideration for use of such an approach is that it focuses on the actual sequence of events which contribute to the combat attrition process. The disadvantage in this technique is the heavy requirement for input data and determination of which processes apply to the generation of variables under the changing conditions of the battlefield. Also, synergistic effects from such aspects as supply and logistics are external to the Bonder paradigm, thus requiring some external means to determine their effect or be ignored.

5. Combat Analysis Model (COMAN)

The COMAN model was developed to fill a need for a more efficient aggregated attrition model that could approximate the output of more time intensive, high resolution models such as CARMONETTE. The basic assumptions within COMAN are similar to those laid out by Bonder. These assumptions are:

- that firepower allocation is dynamic and weapon effectiveness changes as forces move about the battlefield
- the attrition coefficients for firer-target pairs and the probability of a target being unacquired are constants over each time interval, and
- the attrition rate at any point in time is equal to the sum of the individual weapon kill rates.
- each firer-target pair interaction can be considered an independent event and as such each time interval which represents an individual battle is also independent of preceding and succeeding intervals

The approach develops a series of maximum likelihood estimators for weapon kill rates balanced with values for probability that an opposing target will be undetected and prioritization of targets. These factors are then used to determine attrition within each time step in the aggregated simulation. To achieve this end, COMAN and its successors (COMANEX and COMANEX) use input parameters from a high resolution model, based on various force mixes, tactical situations, weapon

characteristics and terrain combinations in order to generate the corresponding attrition coefficients. [Ref. 7: p.174]

The maximum likelihood estimators for attrition coefficients and the probability of a target being undetected are computed for successive time intervals based on only the data relevant to that specific interval. The estimators are defined by analyzing the data from several replications of a high resolution battle which have similar tactical, force and terrain factors, where a battle is defined by firer-target pairings. For each set of battle data (i.e. the observations of each type firer-target pair) a maximum likelihood estimator is calculated and applied to a specific time interval within the aggregate model. This process is then repeated for all the time steps of the aggregate model. By using the maximum likelihood estimators of the parameters, the COMAN attrition rates can be considered as asymptotically unbiased and normally distributed with the smallest possible variance for any unbiased estimator. Applying these estimators throughout the aggregate model should then provide attrition figures that closely approximate those achieved in more time-intensive high resolution models and results extrapolated for forces ranging from battalion to division. Results from comparative test indicated that the difference in casualty assessment between a high resolution model and the COMAN model for identical scenarios were between 1 and 3 percent. [Ref. 4: p.60]

While test data indicated that the methodology is accurate and resulted in extensive time savings over running large scaled high resolution models, there remain additional costs to the model approach that are worth noting. Foremost is the need for an extensive library/file of high resolution combat results covering numerous force mixes, mission postures or tactical situations, and weapon characteristics. Subsequent to this, the high resolution model methods for attrition calculation must be verified and acceptable to the customer of the aggregate simulation. Finally, predictions about units is limited by the scenarios available in the high resolution data runs.

The key to the COMAN methodology lies in the subprogram that determines the time between casualties for the various firer-target groupings. Once this is achieved, the use of the maximum likelihood estimator produces a mathematically sound approximation of attrition for each interval. Since each interval is considered to be independent of the surrounding time steps and only data specific to that time step is considered, the resulting parameter can be considered a valid estimator of combat for that interval. Incorporation of target acquisition and detection probability further

imbeds the dynamics resulting from force movements into the estimating parameter, minimizing any judgemental effects which may have been used in the modelling processes. The end product of COMAN and its embellished successors, is a quantum leap forward when compared to subjective firepower techniques which fail to incorporate the time-distance factor into the combat attrition process.

6. Summary

The different approaches briefly described above account for the basic classes of approaches used to model the combat environment and derive attrition coefficients for use in aggregate models. The remainder of this paper will evaluate representative methodologies from the three categories and determine the effect on model outcomes when using these techniques. External modelling factors which do not directly impact on the attrition process are left as areas for future analysis. Each technique will be examined in a baseline scenario and with a weapon system modification for purposes of sensitivity analysis.

III. MODEL DEVELOPMENT AND EVALUATION

A. OVERVIEW

This chapter examines the specific assumptions, procedures and results obtained when using the subjective FPS, analytical FPS, and Bonder methodologies. An analysis of the techniques' appropriateness as an attrition coefficient generator is included within the discussion of each specific approach. In order to achieve this end, a brief overview of the aggregate models and the baseline scenario used in the analysis of the methods is provided. This will be followed by an analysis and discussion of the three methodologies and their respective outputs when applied in the given model and scenario. In-depth comparative and sensitivity analysis is covered in Chapters IV and V.

B. SIMULATION MODELS

Two simplified aggregated models were developed to facilitate the comparison of effects from using different attrition coefficient generation techniques. Both models are programmed in the APL language which allows efficient handling of attrition coefficient and other data vectors/matrices. The models consist of two modules, one that takes the representative attrition matrices and calculates the associated eigenvalues through the Potential-Antipotential (P-AP) methodology. The output from this module allows assessment of relative value of each weapon based on the M1 Abrams tank. The second module simulates the Lanchester Square Law combat process for the forces involved and tracks changes in weapon-specific force strength throughout the simulation. Both modules are updated at each time step to allow for the loss of combat systems before continuing the simulation. In each model the attrition update intervals are 15 seconds.

The initial model requires input of pregenerated attrition coefficient matrices independent of any methodology. Attrition matrices represent evaluation of the weapon killing potential at intervals of 500 meters. In order to reduce the size of the data files necessary to run the simulation, it is assumed that the attrition process behaves in a linear fashion over each 500 meter interval designated by the input matrices. The second model, a modification of the first, calculates the Bonder attrition coefficients at each time step using Eqn 2.4 and enters the values into the Lanchester

and Potential-Antipotential modules. The specific programs are presented in Appendix A and Appendix B, respectively. The choice to model the Lanchester and P-AP methods into a single program allows a more efficient means to collect outputs. If only the Lanchester simulation is used, the number of surviving systems becomes the sole output and the weapon interrelationships during the battle are lost. The alternative of using only the P-AP model will produce values for single weapon systems in the battle but fail to show how the number of each system vary in the simulation at large. Combining the two modeling approaches and simultaneously viewing the respective outputs provides information to the analyst and military user that lend insight into the dynamics of the battle.

Anomalies particular to the use of probability of kill and other associated weapon characteristic matrices required adjustment to some of the input data. The matrix inversion program used in the P-AP method requires that a value greater than zero be assigned to each firer-target pairing. To meet this model requirement, any firer-target pair which would normally have a value of zero (0) received a value of 10^{-10} . While this value was sufficiently small to be considered as a zero value when weighted against other weapon pairs, the relative weights developed through the eigenvalue process in P-AP produced overinflated weapon values. As new weapons were introduced into an active battle role, several weapon values with magnitudes in excess of 10^7 were achieved. This drastically distorts the casual user's assessment of a weapon's contribution to the battle or its relative worth compared to other systems. This may, in turn, lead to a poor decision for weapon procurement or future force mixture policies. The specific effects caused by this anomaly are addressed in more detail in the respective methodology sections.

A second model aberration was noted regarding the probability of kill matrices (i.e. values were required to be monotonically increasing as ranges decreased). Under the particular conditions of the test scenario this did not become a problem. However, introduction of nonmonotonically increasing values during the model development stages produced a 'reinforcing effect' i.e. units were created during the battle. While this can easily be overcome in larger and more sophisticated models, it is an area that inherently may cause problems if ignored when using the eigenvalue process to determine relative value. As a precautionary measure, the scenario was developed to avoid situations that would cause matrix calculations to produce negative values. Consequently, some artificiality has been introduced into the test and evaluation

process by not allowing conditions which result in decreased killing potential. However, this is a model anomaly and not specific to the generation methodologies under investigation. Consequently, this point is felt to have relevance only to modelers using the P-AP approach.

The final assumption is the choice to only portray the rate of combat or pace of battle as being equal for both sides. This assumption allows the use of a single weapon system as the base for scaling all other systems in all three approaches. In the eigenvalue method, this sets the proportional constants, C_x and C_y , to be equal. Additionally, this assumption allows the use of the same baseline system to be used in the subjective and analytical FPS methods and provides a common foundation for analysis. The disadvantage of the assignment of a single baseline weapon system is that interpreting the value of the weapon system and the role it plays in the battle is hidden by the constant value.

C. SCENARIO

In order to evaluate the three approaches for generating attrition coefficients within the Lanchester and Potential-Antipotential simulation environment, it was necessary to develop a common scenario. It was decided to script a simplified battalion level battle which consisted of five different weapon systems in each force. The forces used are a U.S.-type tank heavy task force (Friendly) and a reinforced Soviet-type motorized rifle company (Threat). The critical assumptions in the scenario are that the threat company is entrenched in prepared defensive positions with the friendly task force deployed tactically across the width of the defensive sector. Distances between opposing weapon systems are averaged based on a constant rate of advance of 200m/min. Movement of threat forces is restricted within the defensive strongpoints and as such considered nonexistent, i.e. stationary throughout the battle. All weapon systems are intervisible but subject to the acquisition parameters of their fire control systems. Terrain and weather provide no restrictions to movement. Artillery units fire at their sustained rates and expended rounds by all weapon systems are subject to their normally prescribed basic loads. M1 tanks are able to fire on the move while TOW armed systems, ITV and M2 Bradley, fire from short halts, simulating tactical overwatch positions. All threat weapons are assumed to be firing from hull defiladed positions but without overhead cover. Counterbattery fire (CB) 122mm howitzers is not considered, with the CB mission passed to the 152mm howitzer assigned to the Division Artillery Group (DAG). Direct fire weapon systems are not

permitted to fire on artillery weapons because of range and normal intervisibility restrictions incident to standard tactical deployment of those systems.

The scenario does not attempt to account for air-ground battle nor to portray all possible weapons found in a tank heavy task force. The simulation is limited to a total of ten systems, based on the assumption that increasing the number of weapon systems does not provide better insight to the general attrition process. Additionally, many of the dynamic interactions such as resupply, barriers, terrain and weather are not developed because they should represent identical scalar multipliers in each case. The target acquisition processes which would act as combat multiplier or modifier in the subjective and analytical firepower approaches is a key factor in the Bonder approach, as noted in Eqn 2.4. However, by treating the time to target acquisition as zero and by assuming that a parallel acquisition process occurs, a consistent target acquisition process is maintained in all three simulations [Ref. 3: p.43].

The force structure used throughout the baseline simulation is shown in Table 4. As previously mention, secondary systems - coaxial machineguns, tank commander .50 Cal and 12.7mm machineguns, or infantry weapons such as the M16 and AK74 or the Dragon and Spigot ATGM, are not included although acknowledged as existing and essential in any full scale simulation model. Critical tactical decisions such as simulated engagement ranges for the 25mm cannon or 73mm smoothbore gun, were made prior to the generation of input tables, thus producing scenario specific results.

TABLE 4
OPPOSING WEAPON SYSTEMS

TF Blue		TM Red	
M1 Abrams	42	T72 Tank	4
M2 Bradley	6	BMP(w/ 73mm)	10
M730 Improved TOW	10	AT5 Spandrel	10
M109A2 155mm How	6	122mm SP How	6
M110A2 203mm How	4	152mm SP How	6

The additional number of AT5 ATGM is a result of consolidating the BMP mounted AT5s into the antitank defense system.

D. SUBJECTIVE FIREPOWER SCORES

Firepower scores for the various weapon systems listed in Table 4 were developed by a group of four U.S. Army officers, grades O3-O4, currently attending the Naval Postgraduate School, and are not meant to reflect U.S. Army approved firepower score values. Assigned scores were based on the assessed evaluation of the individual weapon's overall effectiveness against opposing systems throughout the course of the entire battle. For example, while artillery was considered most effective at long ranges and against infantry in the open, which was not an opposing system in this simulation, its overall lethality against tanks and armored vehicles did not overcome the effectiveness achieved by direct fire weapons inside 3000 meters. Initial firepower scores were based on the M1 tank having a value of 1.0. Evaluation of all other systems were done without regard to the operational mission of the system. Thus, combat multipliers for being in prepared defensive positions were not applied to the FPS or FPI of the units after the initial assessment process. While the use of a combat multiplier of 2.0-3.0 for units in the defense is perfectly acceptable, and a long accepted practice, the use of the multiplier in this process relates more to the pace of battle than the chosen measure of effectiveness used to assign system values. Based on the M1 tank, the subjective firepower scores are:

TABLE 5
SUBJECTIVE FIREPOWER SCORES

M1	=	1.0	T72	=	.95
ITV	=	.5	AT5	=	.5
M2	=	.6	BMP	=	.5
155	=	.15	122	=	.15
203	=	.15	152	=	.15

The corresponding firepower index, using Eqn 2.1 and the associated number of systems from Table 4 yields

$$\text{FPI(attacker)} = 52.1 \quad \text{FPI(defender)} = 15.6$$

The resulting force ratio (attacker/defender) is $52.1 / 15.6 = 3.339$. This indicates that the friendly force (Blue) is over 3.3 times more powerful than the defender (Red).

Applying the force ratio of 3.3 as an attrition ratio in an aggregated model results in the equivalent of reassigning all friendly weapons a value of 3.3 and threat weapon

systems a value of 1.0. Obviously, use of a force ratio, developed from the aggregation of individual systems, does not retain the aspects of the original weapon weights and cannot be used as an attrition coefficient in an aggregated model. By using the force ratio as a representative attrition coefficient for units, individual weapon interaction is ignored, thus producing results that are highly unlikely to occur in a real battle. For example, a pure infantry force with no other weapons than M16s could defeat a tank company. In order to prevent such an occurrence in aggregated models, it is possible to compare the firepower scores of each firer-target pair and provide a relative weight matrix (W_{ij}) for each unit.

The relative weight matrices are produced by using the equation $W_{ij} = \text{FPS}_i / \text{FPS}_j$, where W_{ij} is defined as the relative value of weapon i as compared to weapon j . This provides the associated value of each weapon system against an opposing force system. For example, a T72 tank is worth .95 M1 tanks, but is also worth 1.9 ITVs or 1.583 M2 IFVs. The relative weight matrix (W_{ij}) for the Red force is

TABLE 6
RELATIVE WEIGHT MATRIX

	M1	ITV	M2	155	203
T72	0.95	1.9	1.5833	6.333	6.333
AT5	0.5	1.0	0.8333	3.3333	3.3333
BMP	0.5	1.0	0.8333	3.3333	3.3333
122	0.15	0.3	0.25	1.0	1.0
152	0.15	0.3	0.25	1.0	1.0

The relative weight matrix for the Blue force can be found by taking the inverse of the the respective column values. For example the M1 tank has a corresponding relative weight of 1.05, 2, 2, 6.67 and 6.67 when compared to the T72, AT5, BMP, 122 and 152 weapon systems, respectively.

The associated attrition matrices, which are constant throughout the entire battle simulation because the evaluation of the initial values were assessed based on the overall battle contribution, are approximated by the relative weights. For example, a T72 with a relative weight of 1.9 against an ITV would be expected to kill 1.9 ITVs for every 1 tank lost to an ITV. Extension of this logic to all the system pairs produces attrition matrices A_{ij} and B_{ji} and displayed below in Table 7, where

A_{ij} = the rate at which one firer type j kills one target type i
 B_{ji} = the rate at which one firer type i kills one target type j

TABLE 7
SUBJECTIVE FPS ATTRITION COEFFICIENTS

	A_{ij}				
	T72	AT5	BMP	122	152
M1	0.9500	0.5000	0.5000	0.1500	0.1500
ITV	1.9000	1.0000	1.0000	0.3000	0.3000
M2	1.5833	0.8333	0.8333	0.2500	0.2500
152	6.3333	3.3333	3.3333	1.0000	1.0000
203	6.3333	3.3333	3.3333	1.0000	1.0000

	B_{ji}				
	M1	ITV	M2	152	203
T72	1.0526	0.5300	0.6316	0.1579	0.1579
AT5	2.0000	1.0000	1.2000	0.3000	0.3000
BMP	2.0000	1.0000	1.2000	0.3000	0.3000
122	6.6667	3.3333	4.0000	1.0000	1.0000
152	6.6667	3.3333	4.0000	1.0000	1.0000

Introduction of these attrition matrices into the model resulted in a Blue victory in 6 time steps, or 1.5 minutes. This equates to a ground separation distance between opposing forces of 4850 meters when victory conditions were achieved. This result contradicts the intuitive expectation of combat flow. If these results were accepted, artillery fires, which represent less than 3 percent of the total Blue force firepower, would account for all Red force losses. Additionally, no direct fire weapons would have entered the battle beyond the movement phase. Consequently, an initial conclusion that use of relative weights based on an overall battlefield evaluation fails to account for range and weapon characteristics appears sound. Further, use of such values produces an unrealistic pace of battle.

Reduction of the A_{ij} B_{ji} matrices by a factor of 100 in order to balance the attrition process over all possible time steps resulted in a battle duration of 26 time steps, or 6.5 minutes and a ground separation distance of 3700 meters at battle end. The critical points in the simulated flow of combat are shown in Table 8 .

TABLE 8
POINT OF COMBAT INEFFECTIVENESS

SYSTEM	TIME(min:sec)	RANGE
122mm How(R)	3:30	4300
152mm How(R)	3:30	4300
T72 tank	5:15	3950
BMP IFV(R)	6:30	3700
AT5 ATGM(R)	6:30	3700

The matrices used to compute these figures are fitted to the
the scenario requirements specified in section 3, above and
do not reflect the artillery values noted in Table 7

Even with a reduction of the original matrices by a factor of 100, the intuitive and physical flow of battle remains unacceptable. Direct fire weapons, possessing the preponderance of firepower, are still beyond the range for utilization. The pace of battle remains far too swift. In order to slow the pace of battle within the simulation, any set of external factors such as a target acquisition coefficient or combat posture multiplier can be factored into the original matrices. Reduction of the current matrices by a factor of 2 extended the battle to 6.5 minutes, and a closing distance of 3700 meters. Subsequent rescaling of the matrices by 0.5 and 0.5, an overall equivalent of 0.00125 scaling of the original attrition coefficients, ultimately produced a simulated battle of 21.5 minutes and force separation distances of 700 meters at battle end, a much more intuitive result.

One possible explanation for this high rate of battle lies in the fact that when the subjective firepower scores were developed outside of a range/time dependent function, the firer-target kill rates are uniformly distributed over each time interval. This is the same as assigning values to the weapon systems as if they were always at the point of their maximum effectiveness. This observation holds true for any attrition coefficient computed using a static firepower scoring technique. In order to offset this effect, it is either necessary to develop a firepower score at each step of the battle based on range dependent characteristics or modify the initial attrition coefficients by a time range dependent function. The first choice results in a shifting of techniques from subjective to analytical. The second option modifies the attrition coefficient and is not an

integral part of the generation technique. These points illustrate that the subjective firepower score approach does not produce an attrition coefficient that realistically portrays combat dynamics.

Use of range or time dependent equations in future transformations provides the means to add realism back into the simulation. However, it is important to remember that modification of the attrition coefficients after their generation is a modelling technique and not part of the weighting process discussed previously. Once a firepower score is assigned, it loses the characteristics used to derive the value. Although the score takes into consideration the various attributes of a system, the result is an index used for comparison on a relative scale. Further, it remains to determine which modifiers can rightfully be applied to the original attrition values to account for such a reduction. If factors such as target acquisition, mission posture, and logistic status are included in calculation of individual firepower scores, can these same factors be applied a second time against the aggregated attrition coefficients without biasing the results? Multiple use of a modifier is the same as raising the factor to a power which may result in the violation of any additional linear assumptions made later in the model, or create nonlinear attrition rates where linearity exists.

E. ANALYTICAL FIREPOWER SCORES

1. Crude Analytical FPS

As pointed out in the subjective firepower scoring discussion, one option available to balance the pace of battle is to develop the firepower scores as a function of time or distance from target. One such method considers the ammunition expenditure rates and the probability of firer type i achieving a kill. The prediction of firepower scores is made by using the basic physical characteristics which result in combat casualties. Therefore, one can expect that a more accurate measure of firepower scores can be achieved by using the formulas

$$FPS = \sum (\text{Ammunition Expenditure}) \times (P(\text{kill}))$$

where $P(\text{kill})$ = the average probability of system i killing any system j and

$$FPI = \sum FPS_i \times X_i$$

where X_i = the number of systems of type i

Using these relationships, the FPS and FPI for the individual systems and opposing forces from Table 4 are:

TABLE 9
ANALYTICAL FIREPOWER SCORES

System	P(Kill)	Ammo Expend.		FPS	X_i	FPI
		per System				
M1 Tank	0.35	40		14.0	42	588.0
M2 IFV	0.55	12		6.6	6	39.6
M730 ITV	0.55	12		6.6	10	66.0
M109A2 How	0.03	60		1.8	6	10.8
M110A2 How	0.03	60		1.8	4	7.2
T72 tank	0.70	40		28.0	4	112.0
AT5 ATGM	0.80	10		8.0	10	80.0
BMP IFV	0.40	25		10.0	10	100.0
122mm How	0.03	60		1.8	6	10.8
152mm How	0.03	60		1.8	6	10.8

The FPI for the Blue force equals 711.6 while the FPI for the Red force is 313.6. The overall force ratio for the two forces, $FPI(B)/FPI(R)$ is 2.269.

There are several areas worth noting for this initial phase of developing analytical firepower scores. First, while using only an averaged probability of kill and basic ammunition expenditure figures, the percentage of firepower contributed by each system is relatively unchanged ($\pm 3\%$). The only marked exception was for the T72 tank which accounted for 35 percent of the total Threat firepower, an increase of over 11 percent. Second, the force ratio showed a significant change from the one generated from perceived values of the subjective approach. Finally, the application of a situational factor to determine the average probability of kill and ammunition expenditure appears to lend a more reasonable assessment of the true force strengths in a specified scenario. However, the use of average probability of kill as a constant throughout the simulation and the original problem of applying static force values over a period of time is still present. Therefore, even with the introduction of situational factors directly to ammunition expenditure and probability of kills, it is still necessary to use the relative value scaling method to develop attrition coefficients. Computation of attrition coefficients follows the same procedures used in section C above. Specific values for relative worth, attrition coefficients, battle flow and victory conditions can be found in Appendix C.

Initial analysis indicates that although the method to generate firepower scores appears to be analytically sound and uses only technical data derived from high resolution simulations and/or experience, it is little more than inputting technical data through the subjective firepower approach. Additionally, the pace of battle was significantly changed by the method. A scaling reduction factor of .0025 was required to bring the pace of battle into line with a reasonably expected flow of combat for the given scenario. These observations, coupled with those from the section on subjective firepower scores, indicate that the use of firepower scores for the basis of attrition coefficient computation results in an overestimation of the weapon systems' actual value as a casualty inflictor. Unfortunately, this leaves the modeler two methods for generating firepower scores, but none for computing reliable attrition coefficients for use in aggregate simulations.

2. Range Dependent Analytical FPS

The most apparent shortfall observed in the two firepower score approaches is the use of all encompassing variables to describe the value of a system. A second deficiency is the confusion in transforming firepower scores into attrition coefficients. Therein lies the crux of the problem. In order to understand why the problems resulted and how to alleviate them, it is necessary to return to the definition of firepower scores and attrition coefficients. Restating the previous definition, a firepower score is the relative value of a weapon based on its firepower. The ratio of firepowers scores then represents a dimensionless variable. While an attrition coefficient is defined as the rate at which a single firer i kills a target system j , or

$$a_{ij} = \text{no. of } j \text{ casualties} / ((i \text{ firer}) \times (\text{unit of time}))$$

Since there is no dimensional equivalence between the force ratio value and casualties firer \times time, it is necessary to develop attrition coefficients as a function of those variables that contribute to production of casualties.

The method used to illustrate this approach, and considered as the principal example of the analytical firepower technique is:

$$A_{ij} = \alpha_{ij} \times v_j \times P_{ssk} \quad (\text{eqn 3.1})$$

where

α_{ij}	=	acquisition rate of target i by firer j
v_j	=	the system rate of fire for firer j
P_{ssk}	=	the probability of a single shot kill

Acquisition rates were computed based on the following criteria:

- a concept of the flow of battle as forces close
- multiple weapon engagement of single targets could occur
- systems in overwatch positions versus those on the move
- target acquisition capability is based on the fire control system of weapon.

System rates of fire were based on data from U.S. Army FM 101-10-1, RB 101-999, unclassified documents on U.S. and Soviet weapon systems and military experience. Rates of fire were averaged based on the closing speed of the attacking force, expected target acquisition rates and the basic loads of ammunition carried by each weapon system. This precluded the 'unlimited resupply' syndrome often found in first generation aggregated simulation models. Probabilities of single shot kills were based on conditioning the probability of kill on the probability of a hit for a given range window. For simplicity, attrition coefficient matrices were established for 500 meter windows from 5000 meters to 0 meters. Computation of coefficients within each 500 meter window was achieved by dividing the interval into 10 independent sections of 50 meters each equal to one 15 second time step. Assuming that linearity existed within the interval the subinterval values were then calculated by interpolation. Use of piecewise linear interpolation within each interval allowed the assumption that any existing nonlinearity in the probability of kill would be retained over the most of the range of combat.

At this point a brief review of the input factors is needed to eliminate possible misinterpretation of the variables used in this approach. The acquisition factor for a system is the percentage of target type i that firer j can acquire. This factor is influenced by terrain, tactics and weapon characteristics. For example, artillery, which uses forward observers, front line forces, aerial observation and electronic warfare input may have an acquisition factor of 1.0 for all enemy weapon systems. This would indicate that it can acquire and engage 100 percent of opposing target type j . A tank may only have a 0.4 acquisition rate (indicating that it can acquire and engage up to 40 percent of all enemy systems) because of his sector of fire and equipment constraints limiting his field of vision. Further, the target acquisition rate should not be confused with the time to target acquisition (t_a) used later in the Bonder technique. Acquisition time pertains to the period that is required to find the next target for engagement, whereas the acquisition is the percentage of targets of type i that a firing system identifies and engages. The second variable, system rate of fire is based on the weapon systems sustained rate of fire and adjusted to fit the time steps

used in the model. Therefore, an artillery piece with a sustained rate of fire of 6 rounds per minute would have a rate of fire of 1.5 in a model with 15 second time steps. Finally, the probability of a single shot kill considers the systems combat mission (attack vs. defend) and conditions the probability of kill on the probability that a hit occurs, i.e. the hit will result in a combat kill.

Using Equation 3.1, attrition matrices for the Red and Blue forces are generated. They are then adjusted to allow for the specific scenario conditions such as no direct fire engagement of artillery units. Target acquisition factors based on tactical consideration for sectors of fire are applied to the direct fire systems, reducing the number of systems a single weapon system can engage and slowing the pace of battle. Application of these factors results in a series of attrition coefficient matrices that reflect the conceptualized flow of battle and are considered to be scenario specific.

Based on this approach the battle is partitioned or visualized into three sectors; the indirect fire, the long-range fire, and the close-in fire zones. The indirect fire sector is dominated by the artillery fires with direct fire systems outside of their engagement ranges. A representative attrition matrix for the 5000-4500 meter range window is shown below.

TABLE 10
INDIRECT FIRE ZONE ATTRITION MATRIX

Red Force Attrition Matrix for 5000-4500 meters					
	T72	AT5	122	BMP	152
M1	.00000	.00000	.00050	.00000	.00019
ITV	.00000	.00000	.00350	.00000	.00134
M2	.00000	.00000	.00250	.00000	.00096
155	.00000	.00000	.00000	.00000	.00278
203	.00000	.00000	.00000	.00000	.00278

The long-range fire zone marks the points where the guided weapon munitions and maximum line-of-sight fires enter the battle. This covers the area between 4000 and 2000 meters. Tank firing rates are low and antitank guided missiles (ATGM) are the primary killers. Artillery fires remain constant but their contribution to the battle as a casualty inflictor is overcome by the accuracy and kill probabilities of the ATGM systems. An example of the attrition matrix corresponding to the changed battlefield condition at the 2500-2000 meter window is

TABLE 11
LONG-RANGE FIRE ZONE ATTRITION MATRIX

Red Force Attrition Matrix at 2500-2000 meters					
	T72	AT5	122	BMP	155
M1	.00000	.08880	.00050	.00000	.00019
ITV	.04000	.07040	.00350	.00000	.00134
M2	.04800	.06800	.00250	.00000	.00096
155	.00000	.00000	.00000	.00000	.00278
203	.00000	.00000	.00000	.00000	.00278

The final phase of the battle is the close-in fire zone. In this area, the flat trajectory, unguided direct fire weapon systems with high rates of fire, i.e. tanks and IFV cannons, dominate the battle. The ATGMs systems whose previously dominance was based on their accuracy and lethality cannot match the values generated by the higher firing rate and increasing $p(\text{kill}/\text{hit})$ of the direct fire systems. Artillery fires are concentrated by the defender and shifted by the attacker but still do not have the lethality of the direct fire weapons. A final attrition matrix for the Red forces at the 500-0 meter window is

TABLE 12
CLOSE-IN FIRE ZONE ATTRITION MATRIX

Red Force Attrition Matrix at 500-0 meters					
	T72	AT5	122	BMP	155
M1	.76500	.00000	.00050	.06000	.00019
ITV	.55200	.00000	.00350	.51000	.00134
M2	.54000	.00000	.00250	.60000	.00096
155	.00000	.00000	.00000	.00000	.00278
203	.00000	.00000	.00000	.00000	.00278

A full listing of the attrition matrices used for the Blue and Red forces, at 500 meter intervals can be found in Appendix D.

Before introduction of the new range dependent attrition matrices into the simulation model, a cursory comparison with the crude analytical FPS matrices was undertaken. This revealed several inconsistencies which probably contributed to the

initially high combat rates observed in the unmodified coefficient runs. Using the range dependent analytical coefficients as a base, the values derived from the crude FPS approach approximated the Threat tanks as operating in the 3500-3000 meter range; the AT5 ATGMs in the 4000-3500 meter range; and the BMP IFV in the 2000-1500 meter range window. Artillery was between 1.4 and 2.0 times more effective than the range dependent generation coefficients at all ranges. In each case, the system was operating near its maximum range and not at the median range as would be expected based on an average probability of kill. This would account for the notably high pace of battle observed. This indicates that the crude analytical approach, like the subjective approach when used as surrogates for actual attrition rates, tends to overestimate the individual weapon values.

After applying the new matrices to the simulation, model output tended to support the supposition that the crude firepower approach overestimated the weapon system killing potential. Duration of combat runs was nearly identical without the need to apply an unspecified modification factor to align the pace of battle with the initial concept for the flow of battle. Combat casualty/fallout effects were also more realistic with the majority of kills occurring inside 3000 meters. A comparison of the flow of weapon eliminations in both analytical methods is provided in Table 13 .

TABLE 13
ATTRITION IN ANALYTICAL FPS MODELS

Crude Analytical		Range Dependent	
System	Time/Range	System	Time/Range
AT5	1:15/ 4850	M2	11:15/ 2750
BMP	16:15/ 1750	ITV	12:30/ 2500
T72	18:15/ 1350	T72	14:15/ 2150
—	—/ —	AT5	15:15/ 1950
—	—/ —	BMP	18:00/ 1400

Under the crude analytical approach the Red force ATGMs were eliminated in battle after 1 minute and 15 seconds or before moving 150 meters of ground distance after the start of the battle. Under the model/scenario conditions, these kills could only be achieved by the artillery fires. This contradicts the basic concept for the flow of battle within the model. The points for total system attrition of the BMP and T72

tank systems were considered acceptable. The final weapon in each system was eliminated when the Blue forces reach the 1750 and 1350 meter points, respectively. Under the crude analytical approach, no Blue force system was eliminated from battle. Output from the range dependent analytical FPS method produced total system losses on both the Blue and Red forces. Blue force M2 IFVs were eliminated from combat at the 2750 meters after being subjected to the combined fires of artillery, tanks and ATGMs. The ITV system was likewise eliminated by the 2500 meter mark under the same fires as the M2. The remaining systems on the Red force were then eliminated by the surviving Blue force M1 tank and artillery fire.

The significance of this modelling approach is that the factors used to develop the attrition coefficients are adjusted for the range between the forces at each step and therefore a new firepower score is generated for each specific time interval rather than for the whole battle. Unlike the previous scoring methods, it is no longer necessary to use the ratio of firepower scores as a surrogate for the attrition coefficients. By using the range/time dependent values, we have moved away from the dimension problem that plagued the earlier methodologies. However, before accepting this approach as being an appropriate technique for production of attrition coefficients, it should undergo dimensional analysis. As previously discussed, the dimension for an attrition coefficient is casualties/[(firer)x(time)]. Dimensional analysis of the range dependent firepower score equation indicate that the resulting values are casualties/[(system)x(time)], which meets the Lanchester Square Law definition. Therefore, by selecting variables that are functions (discrete or continuous) of time, range and focusing on the actual process of the killing cycle, the technique has produced a weapon value (FPS) which can be used directly as an attrition coefficient. The results produced in the Lanchester Square Law simulation proved to be a more realistic and acceptable portrayal of combat.

F. BONDER ATTRITION COEFFICIENTS

The Bonder approach can be considered as a more sophisticated technique for generating attrition coefficients than the Range-Dependent Analytical FPS method (RDAFPS). Whereas the previous methodologies wavered between trying to define firepower scores and attrition coefficients, and attempted to equate them. The Bonder technique deals directly with the generation of coefficients. Determination of relative weapon values is computed outside of the Bonder equation by subroutines such as provided by the P-AP model. As in the RDAFPS approach, the process by which a

system kills another system defines variables used within the Bonder equation. Review of Eqn. 2.4 indicates an increase in the number of factors considered as well as a more precise definition of the variables. What is hidden from the casual user is the various functions, discrete and continuous, that are used to compute these values (either internal or external to the model). Thus, the technique is somewhere between a high resolution modelling approach and the oversimplified aggregation models previously discussed.

In order to maintain maximum consistency between the model runs, inputs were duplicated wherever possible. The probability of a kill conditioned on the event that a hit occurred remained unchanged from the values used in the RDAFPS method. Likewise, tactics and firing rates were kept consistent. Other assumptions made about the input values were:

- times to fire a round after a hit and after a miss are equal ($t_m = t_m$)
- time to acquire the next target following a successful engagement (kill) was 0 based on the parallel acquisition process described by Taylor [Ref. 3: p.39]
- $P(h|h) = P(h|m)$
- multiple weapon system platforms (BMP and M2) would first use their long range weapons (ATGM) then switch entirely to cannon fire at a specified range
- $P(h|h)$ was based on all Blue forces considered as moving, in the open and presenting frontal views only. All Red forces were considered in hull defiladed positions.

A complete breakout of the input values for the opposing forces is presented in Appendix E.

In the model design structure, the Bonder attrition coefficient matrices are generated at each time step. This precludes any pre-simulation analysis of the coefficient matrices with the RDAFPS values. However, initial expectations were that there should be notable differences in the output. This expectation was based primarily on the use of more clearly defined variables such as probability of hit and the individual times used in determining the weapon firing cycle. While a change in coefficient values was expected, the direction of change (increase or decrease) was not predictable before actual computation.

The direction of change when compared to the RDAFPS method cannot be generalized (i.e. always result in a larger or smaller attrition coefficient) because the use of an average firing rate in the analytical approach may lead to an over- or underestimation of the firing cycle length for a given time step. Since the Bonder

equation provides a specific point estimate on the attrition curve line rather than an average value across the interval, the magnitude of change displayed between the two techniques is expected to be greatest when a nonlinear (concave or convex) attrition process is involved. Therefore, if any of the combat processes are considered nonlinear, the Bonder approach should provide a better estimate of the attrition coefficients than the RDAFPS method.

For comparative purposes the Bonder generated attrition matrices for the three basic fire zones are shown in Table 14 . At the 4500 meter range, the direct fire systems remained the same as expected when outside their engagement ranges. The artillery systems displayed a split behavior with the 122mm system exhibiting slightly lower values and the 152mm system having nearly twice the previous value. At the middle and lower end ranges the direct fire systems were in all cases smaller. The decreased value of the attrition coefficients computed using the Bonder equation will result in an increase in the duration of combat. However, the flow of system attrition is dictated by the relative contribution of each system to the battle. Therefore, even when there is a reduction in the coefficient value on the Red force, there is a corresponding change in the Blue force values and it is their relative magnitudes that drive the simulation. The resulting critical points, and system status can be found in Appendix F. A summary of the system termination points is shown in Table 15 .

G. SUMMARY

Three approaches for developing attrition coefficients have been discussed and evaluated for use in an aggregated model. The subjective firepower score approach provided surrogate attrition coefficients based on the relative weights of the various weapon systems. This created the problem of representing a casualty rate with a dimensionless ratio. While there exists numerous methods for creating firepower scores, development of relative worth matrices for several methods such as the AWC resulted in a high pace of battle. This may lead to a false interpretation about indirect fire weapons lethality, as well as optimal force mixtures. Interpretability of direct fire systems may also be skewed and conclusions of greater lethality at longer ranges may be drawn, although contrary to experience. The subjective approach exhibited a tendency to overestimate the attrition rates without extensive external modification. This could lead the user into the problem of trying to fit data to the concept of the battle (DePuy's QJM methodology). This 'cart before the horse' approach can lead to misinterpretation of the system's contribution to specific force structures as well as its effectiveness against opposing systems.

TABLE 14
BONDER EQUATION GENERATED ATTRITION MATRICES

Red Force Attrition Matrix at 4500 meters

	T72	AT5	BMP	122	152
M1	.000000	.000000	.000000	.000450	.000346
ITV	.000000	.000000	.000000	.003150	.002419
M2	.000000	.000000	.000000	.002250	.001728
155	.000000	.000000	.000000	.000000	.005004
203	.000000	.000000	.000000	.000000	.005004

Red Force Attrition Matrix at 2000 meters

	T72	AT5	BMP	122	152
M1	.001294	.009606	.000000	.000500	.000384
ITV	.004961	.010542	.000000	.003500	.002688
M2	.002731	.001424	.000000	.002500	.001920
155	.000000	.000000	.000000	.000000	.005560
203	.000000	.000000	.000000	.000000	.005560

Red Force Attrition Matrix at 0 meters

	T72	AT5	BMP	122	152
M1	.095132	.013512	.004837	.000500	.000384
ITV	.062834	.013212	.030240	.003500	.002688
M2	.094588	.013993	.035934	.002500	.001920
155	.000000	.000000	.000000	.000000	.005560
203	.000000	.000000	.000000	.000000	.005560

TABLE 15
ATTRITION IN BONDER COEFFICIENT MODEL

System	Time/Range
T72	14:30/2100
BMP	15:15/1950
AT5	21:30/ 700

The analytical firepower approach introduced the concept of applying the fundamental aspects of the killing process by assignment of a value for each system. Using a single average probability of kill and average operational ammunition expenditure rates produced the number of kills round for each system. However, the comparative evaluation of the opposing systems created the same dimensional problem exhibited by the subjective FPS method. Further development of the analytical approach incorporated range dependent inputs. This replaced the previous static values with a series of time/range dependent scores and reduced the problem of over or underestimation of the weapon kill potential over the duration of the battle.

Of particular note is the analytical FPS approach's movement away from using the contribution of a system relative to the other systems and focusing on the means by which a firer kills a target. Factors that affect or define the actual activities of attrition should be included in the generation technique as it would be in a high resolution model. Factors that may enhance or degrade the effectiveness of the firer-target interaction should be used as external or after-the-fact modifiers. For example, weather degrades target acquisition in a tank-tank battle but not the actual $p(k|h)$, therefore it should not be incorporated in the coefficient generation process. Attrition is a clearly definable process and should remain that way when modeled at any level.

The range dependent analytical firepower score approach is a simple, straightforward methodology. It is easy to understand and compute. Its main detractor is its reliance on the average rate of fire. Removal from range dependence ignores the critical factor that produced the major improvement over the previous approaches. However, the techniques provides combat results that were more realistic and correspond to the conceptualized view of the modern battlefield without violating the tactics and physical laws governing the weapon systems.

The final technique, the Bonder approach, developed attrition coefficients directly. As previously noted, by increasing the number of input variables the approach acts like a sophisticated RDAFPS technique. Values for the variables are generated through the use of discrete or continuous functions, either internal or external to the model. At each step a new attrition coefficient is generated and applied only to that time period. The method allows all input parameters to vary with time or distance and leaves external weighting factors such as weather and terrain to be incorporated outside the actual coefficient equation. The pace and duration of battle are set by the firing cycles of the weapons rather than the expected firing rates.

Further, the probability of killing a target and the conditioning aspects that consider $P(h|h)$ and $P(h|m)$ increase the models credibility by recognizing that multiple firings may occur before a kill results. Previous methods basically ran on an 'if you fire enough rounds you will achieve the desired number of kills' summation approach.

Regardless of the processes used in these methods, equivalent results should be expected if all the defining characteristics of firepower scores, indices, ratios and attrition coefficients are met. A firepower score regardless of how generated is still a firepower score. A ratio of firepower scores is a dimensionless value and not a casualty rate. Likewise, the use of more input variables to generate proper attrition coefficients, if within the actual firer-target killing cycle, should provide better output. Further, the use of noncontributing factors in the actual calculation processes such as the Bonder equation, tend to dilute the final values.

The use of subjective FPS as a surrogate attrition coefficient is not recommended for the reason enumerated in this chapter. The choice between the simpler RDAFPS and Bonder approach requires more indepth consideration. As both provide a reasonably outcome under a Lanchester Square Law combat simulation, it would seem logical to save time and effort by using the RDAFPS technique. However, the answer lies in the end use to which the simulation output are to be used. Analysis of the output from the Potential-Antipotential model provides additional insight about these approaches and what they tell the analyst and customer. Chapter IV and V examine Potential-Antipotential outputs and analyze the importance of these values in light of the customer decision process.

IV. MODEL OUTPUT ANALYSIS

A. OVERVIEW

The results from combat simulations provide varied information to different clients. Understanding exactly what the outcome of the battle is becomes important to a commander in the field who makes decisions on force organization and weapon support allocations. The Lanchester Square Law model allows the decision maker to consider the critical numbers necessary for theoretical combat success. In the case of the force modernizer, the relative value or effectiveness of one system compared to another is the critical issue. The Potential-Antipotential model provides the user with the desired relative information. However, the paramount goal of any simulation is to allow an understanding of what happens in the battle and why. Thus, a combination of both model outputs meets either users' needs while providing additional information necessary for sound decision making.

To this end, this analysis examines the overall aspects of the simulation output as interpreted through the use of both models. This will allow explanation of possible cause and effect relationships that are not intuitively obvious. Following analysis of the outputs generated by the different coefficient generation techniques, the effects on model outputs caused by changes in weapon system characteristics are examined. A specific case for improvement to the 25mm cannon system is used as the focal point for this investigation.

B. MODEL OUTPUT ANALYSIS

1. Overall Force Ratios

Remembering that this study is directed toward the effect of attrition coefficient generation methodologies in aggregated models, the obvious starting point for the analysis is the overall force ratios that result from the use of the three methodologies. The force ratio at each time step of the battle is presented in Figure 4.1 below.

As expected, the subjective firepower score ratios are constant throughout the battle. Direct interpretation of this output would indicate that the opposing force systems are equally balanced throughout the battle. This is contrary to the specific scenario used in the simulation. Input parameters were such that an advantage to the

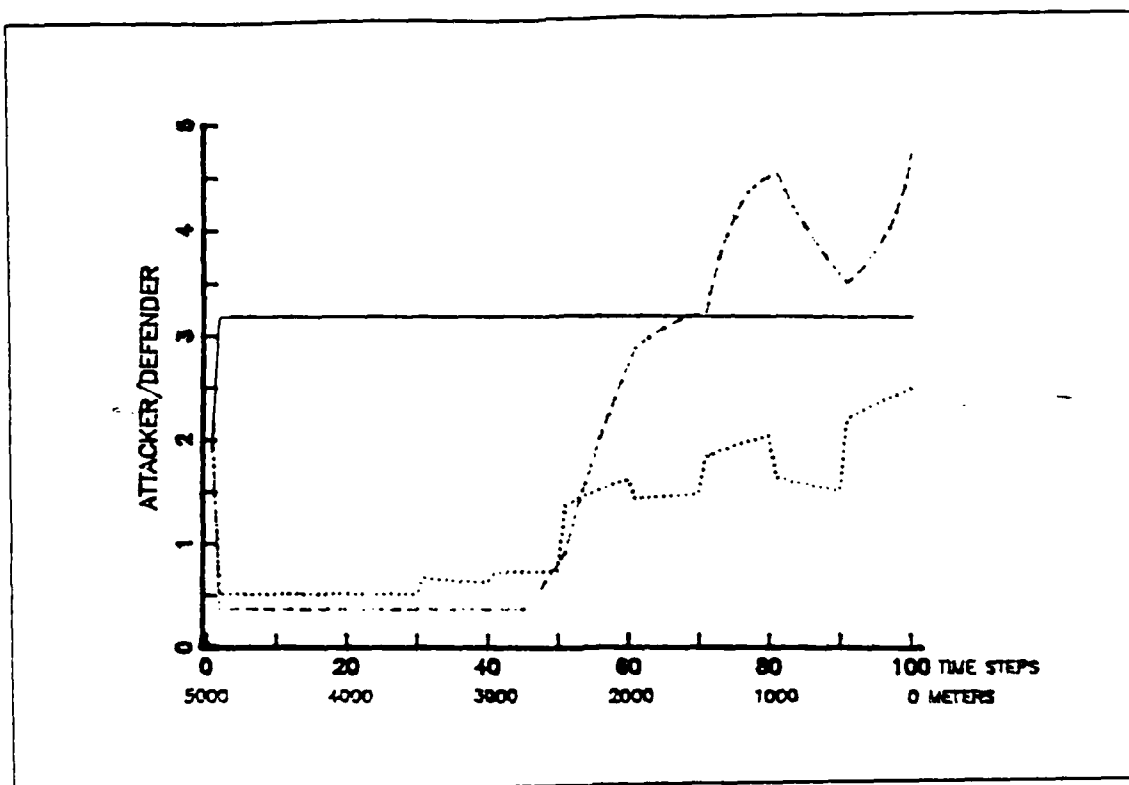


Figure 4.1 Force Ratios.

defender would exist in the indirect fire and early long-range fire zones. Red force artillery firing rates and number of systems, as well as longer ranges for the ATGM system, provided a small but identifiable advantage to the defender. As the battle progressed, the introduction of the M1 tank would then shift the force ratio back to the Blue side. However, this is not reflected by the subjective force ratio plot.

The analytical FPS and Bonder simulation outputs present a significant contrast to the subjective force ratio plot. Both systems show an immediate drop in the attacker/defender ratio, indicative of the advantage of a dug-in enemy that possesses a slight artillery and ATGM advantage. The ratio tends to remain constant until counterbattery attrition effects and increased effectiveness of antitank guided missile fires begin to accumulate. At the 10 minute (3000 meter range) interval, both ratios begin to shift from the defender to the attacker. This corresponds directly to the introduction of the M1 tank into an active role in the combat attrition process and its advantage of longer range fires over the opposing T72 tanks. This increasing trend continues throughout most of the battle. The decreasing values observed near the 500

meter interval marks are attributed to the nonlinearity of the $P(k|h)$ values and the linear interpolation methodology used to assign separate interval values. The significant decline in the Bonder ratio relates to the shifting of weapon systems from the TOW to the 25mm cannon which requires shorter ranges to achieve the 90% or higher lethality of an AT missile.

Of interest is the Bonder ratio's decrease near the direct fire battle terminating (700 meter) point. This results from the continuation of the indirect fire battle, the lack of Red systems to offset the values of the remaining Blue tanks and the decreased value of the BMP systems when switching from the AT5 to 73mm gun system. While the corresponding force ratio patterns are similar between the analytical and Bonder techniques, the magnitudes are clearly larger in the Bonder case after the 2500 meter point. In Chapter III, it was noted that while the coefficients generated by the Bonder equation were smaller, the relative changes in the magnitudes of the various firer-target pairs did not necessarily follow this pattern. Therefore, while it was possible to see a reduction in a coefficient value from 1.0 to 0.8 for weapon A and to 0.6 for opposing weapon B, the relevant change was that weapon A's relative weight compared to B had improved by 1.33. It is this change that accounts for the differences in the magnitudes seen in Figure 4.1.

2. Individual System Analysis

Prior to individual system analysis, it is necessary to restate two effects that are caused by the specific assumptions used in the Potential-Antipotential model. The first is the setting of the value of the M1 Tank to 1.0, as the basis for comparison of other systems. The M1 retains this value throughout all the simulations. Therefore, regardless of the various attrition coefficient values that are computed for the M1, it will always have the singular value output from the P-AP model while other systems change relative to the M1. Consequently, M1 analysis is limited to the Lanchester model outputs. The second aberration is a result of the use of matrix operations which preclude the assignment of zero (0) to the individual attrition coefficients. This manifests itself in the P-AP output in the form of extremely high magnitudes of value for the indirect fire systems as systems with initial values of 10^{-10} enter the battle and are compared with values in the 10^{-3} range. This transition results in value spikes which dominate the normal system values. In order to analyze the values not affected by this system introduced anomaly, the extreme values are deleted from the figures in order to more closely track the unbiased values during the remainder of the simulation.

a. M1 Tank

Evaluation of the three approaches and their effect on the M1 Tank force output indicates a significant difference exists between each method. Previous analysis of the combat attrition flow (Chapter III and Appendices D and F) indicated that different battle flows exist. This is more clearly demonstrated in Figure 4.2. The subjective method produces a flat attrition curve between the initial number of tanks (42) and the surviving quantity of 41.3 indicating an extremely low combat loss level. This may also be interpreted that the M1 survivability profile against the combined enemy systems is high. The analytical approach shows tank strength decreases from 42.0 to 26.8 tanks at simulation end, while the Bonder technique produces losses closer to the subjective approach with a final tank force of 39.6.

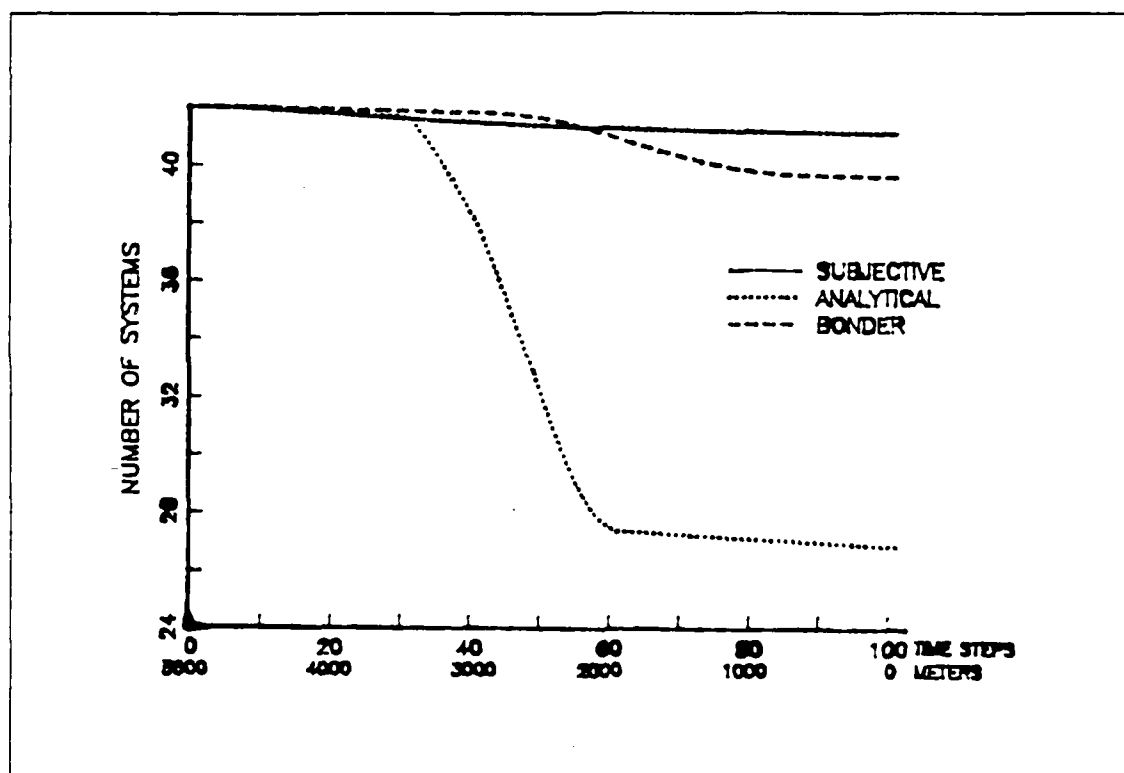


Figure 4.2 Lanchester Model Combat Flow - M1 Tank.

The subjective approach output indicated that little or no loss of tank systems occurred over the entire battle as did the Bonder methodology. This leaves both techniques suspect in their ability to portray the battle. The analytical technique

produced results consistent with tactical expectations indicating heavier losses in the long-range fire and close-in combat zones. Losses decrease to nearly zero once the opposing AT and tank systems have been eliminated. Of the three alternatives, the analytical approach provides the most realistic and acceptable output for the field commander, force modernizer, and analyst.

b. Improved TOW (ITV)

Attrition flow rates in the Lanchester model resulted in the Bonder output shifting toward the analytical FPS system attrition flows. The subjective firepower score again produced a relatively flat attrition curve with values ranging from 10.0 down to 8.7. The analytical simulation resulted in a total loss of ITV systems by the 2500 meter range. The attrition flow plots (see Figure 4.3) indicate that the major losses occurred from the 36th through 50th time steps or the 3200-2500 meter zone. This corresponds directly to the zone where the AT5 ATGM would dominate the battle for the Red forces. Coupled with the higher vulnerability of the ITV's lightly armored carrier and operating in open terrain, these results are tactically acceptable.

The Bonder approach produces an attrition flow that is between the alternative methodologies. Combat losses are heaviest in the long-range fire zone and lessen slightly upon entering the close-in battle area. Following termination of the direct fire ground battle, the system continues to exhibit attrition due to indirect fires, a reality in combat, which is not reflected by the subjective approach.

Analysis within the P-AP environment allows the user to see the relative contribution of the ITV during the entire battle and its relative weight based on the M1 tank. The subjective technique value by definition will remain constant throughout the battle as indicated by the flat plot in Figure 4.4.

In the analytical method, the value remains equal to the M1 up to the 3500 meter range separation mark. This is consistent with military logic as neither system is capable of engaging an opposing system. At the 3500 meter point, the value of the ITV assumes unrealistically large values based on the earlier discussed artificiality required by matrix operations in the model. Once actual computed attrition coefficients for the other systems are introduced into the model, the ITV takes on a value of six times the value of a M1 tank system. This is intuitively acceptable as the longer range fires and high lethality make the ITV the dominant killer. The magnitude of the difference is another question which can be debated indefinitely since it is the end product of a series of nonstandardized input variables. The system value continues

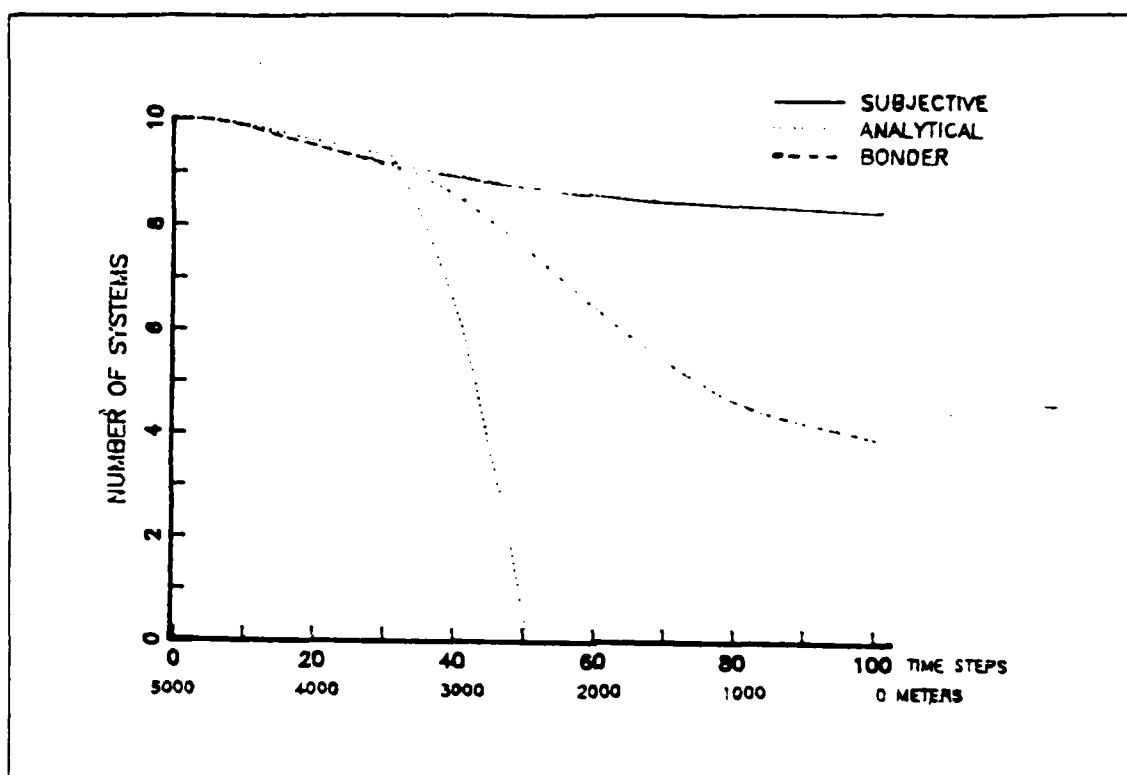


Figure 4.3 Lanchester Model - ITV Combat Flow.

to drop sharply as the tank systems begin to dominate the battle through higher firing rates and steadily improving $P(\text{kill})$. By 1500 meters, the general area of the tank maximum effectiveness, an ITV system has less value than a M1 tank. Its value continues to fall as IFV cannon fire increases and the minimum range requirements preclude further missile launch. Overall, the analytical approach tends to reflect the flow of battle. The resulting system values are consistent with modern tactics and experience which recognizes the importance of ATGM fire as a long-range killer but of minimal value in close combat.

The Bonder output parallels the analytical process when analyzing the ITV's value in the three fire zones. Beyond the maximum range, the system holds a relative value of 1.0. However, where the analytical output exhibited extremely large values for a 500 meter window based on artificial attrition values, the Bonder technique produced usable coefficients at all intervals. Thus the results provide the analyst with a much clearer picture of the value of the ITV systems. Starting from 3500 meters, it can be seen that there is a steep increase in the system's value until it reaches 3.0 at

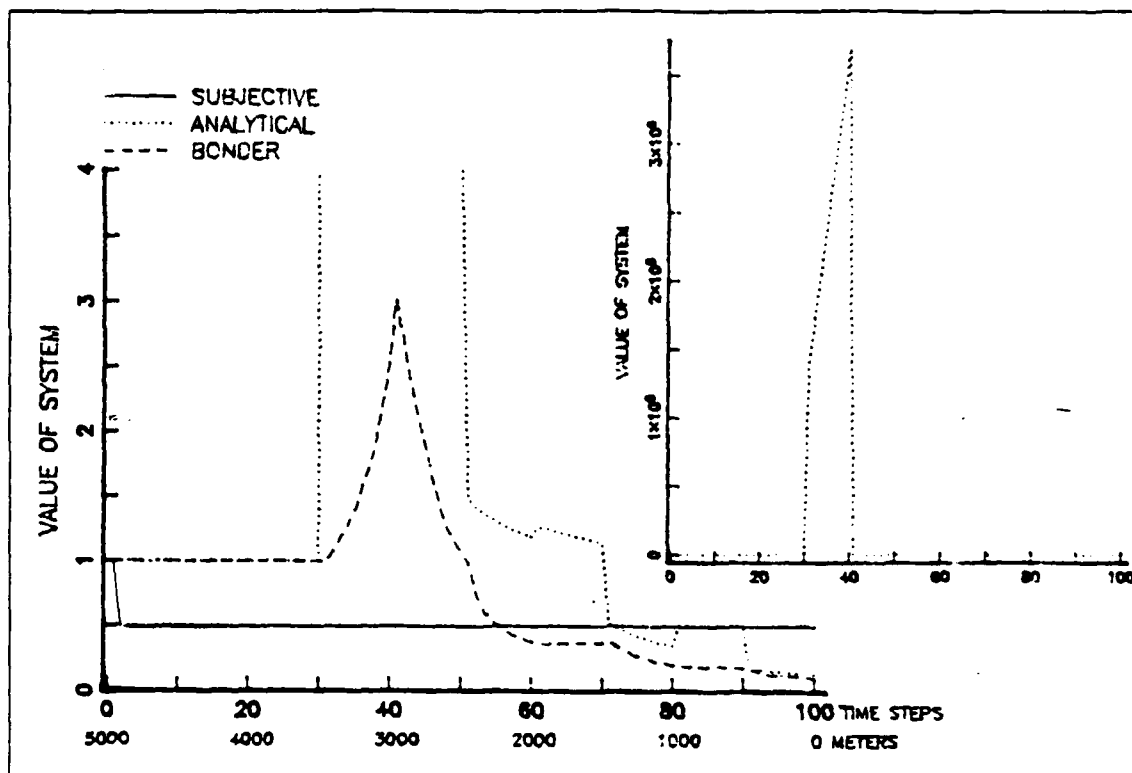


Figure 4.4 Potential-Antipotential Model - ITV System Values.

3000 meters. This corresponds to the point where ATGM accuracy ($P(\text{hit})$) begins to level off. The value of the system then decreases as the remaining systems enter the simulation. At 2500 meters, the system has a smaller value than the M1 tank and gradually decreases to the same point as the analytical approach.

Initial indications suggest that the Bonder method provides the analyst and user a better approach for analyzing the value of a system by providing a continuous string of values from simulation start to end that reflect realistic weapon values. Like the previous technique, the values are tactically acceptable, suggesting that the ITV is of greatest value at the 2500-3500 meter ranges before tanks become more valuable to the commander in his scheme of force organization and conduct of battle.

c. M2 Bradley IFV

The Lanchester and P-AP models produced nearly identically shaped output plots for the M2 and ITV systems. These results are not surprising because the M2 IFV is a multi-weapon platform with the improved TOW as its long-range fire weapon and the 25mm for close-in combat. Beyond the starting force sizes, the tactical

considerations governing the systems are identical until inside 1000 meters and therefore output is expected to be highly correlated.

In the Lancaster simulation, Figure 4.5, the subjective approach produced a nearly linear attrition, with system losses being limited to one system. The analytical method produced output similar to the ITV with high rates of loss once the system entered the long-range fire zone. As with the ITV, the analytical method output indicated that the system would be eliminated from combat near the 2750 meter range mark. The Bonder output continued to fall between the previous two results, displaying increasing attrition levels as the systems neared opposing firers. This is more consistent with combat experience that supports the concept of increased system lethality with diminishing range.

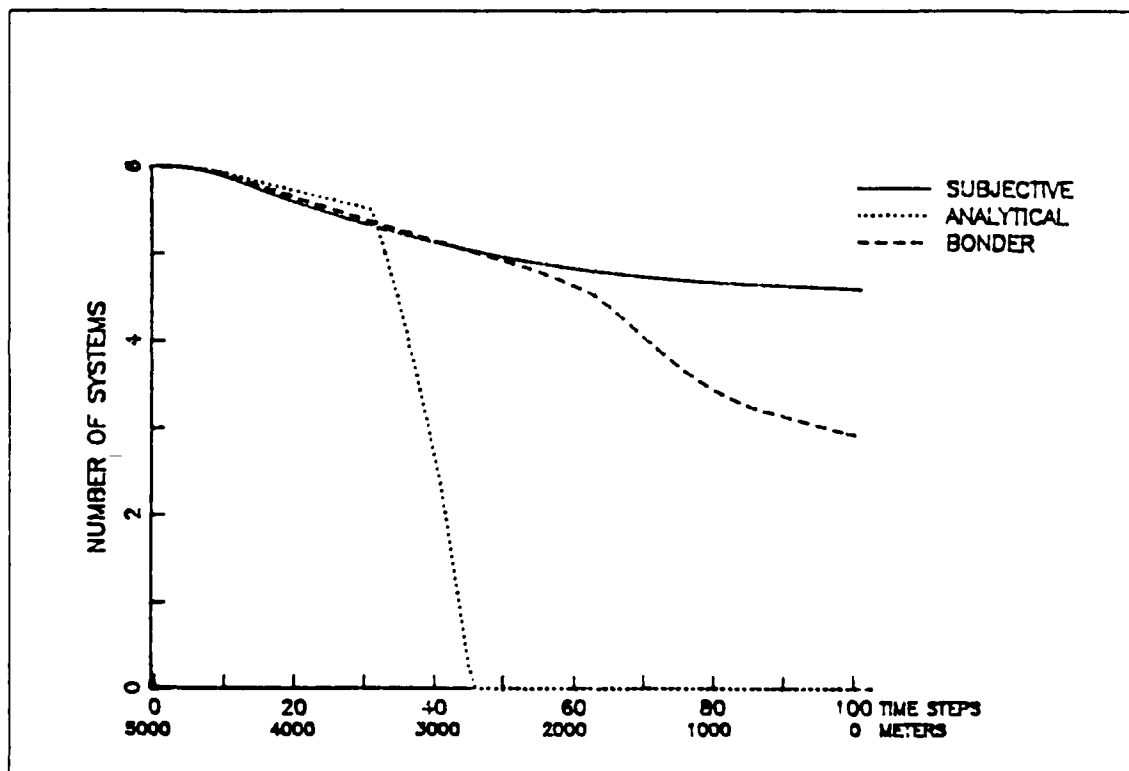


Figure 4.5 Lancaster Model - M2 IFV Combat Flow.

As previously noted, the P-AP model produced results that closely correlate to those noted for the ITV. The subjective firepower output corresponds with the expected fixed value inherent within the method's relative weighting of system values.

The analytical output likewise exhibited the tendency to radically increase in value when the system first enters the simulation as an active killer. This loss of believable information as to system value from the 3500-3000 meter area continues to hinder the analyst and user from reaching any conclusion on the true effectiveness of the system.

The Bonder approach output (see Figure 4.6) allows continuous analysis by the client/analyst unlike either FPS method. Additionally, higher system values were achieved by the systems at several places in the battle. The first point of note is the systems maximum value (6.0). While an initial argument of error may be tendered based on the contention that both the ITV and M2 are using the same weapon, it must be remembered that the P-AP considers more than just killing potential. Each system is evaluated based on its capability to kill opposing systems and be killed by those systems. In this case, the lower level of vulnerability of the M2 compared to the ITV accounts for the increase in system value. The second point of departure from the ITV output was a positive increase near the final battle termination point(500 meters). The resulting increase indicates that the M2 would be considered nearly equal in value to an M1 tank at this range.

The increase in the system value of an M2 can be attributed to the increased lethality of the 25mm cannon in the close-in battle and the notably higher rates of fire. The system's firing cycle, muzzle velocity and P(hit) either match or exceed those of the M1 tank, with only the P(k/h) values preventing the M2 from being of greater value than the M1 tank inside 500 meters. This particular point will be investigated further in the section on model sensitivity when the 25mm weapon characteristics are modified.

At this point of the analysis, several patterns are beginning to develop. First, the Bonder method tends to provide a more balance picture of the battlefield in both model outputs. Additionally, trends in the relative values given to the various weapon systems seem to fit the tactical and doctrinal parameters better than the other alternatives. The subjective approach tends to exhibit lower attrition rates in the Lanchester simulation and provides no relative value information from the P-AP runs that could not have been determined before the computer runs. The analytical firepower results tend toward higher attrition rates and long periods with constant attrition. Finally, the relative values for the weapon systems under the analytical approach are predominantly higher than for the other methodologies.

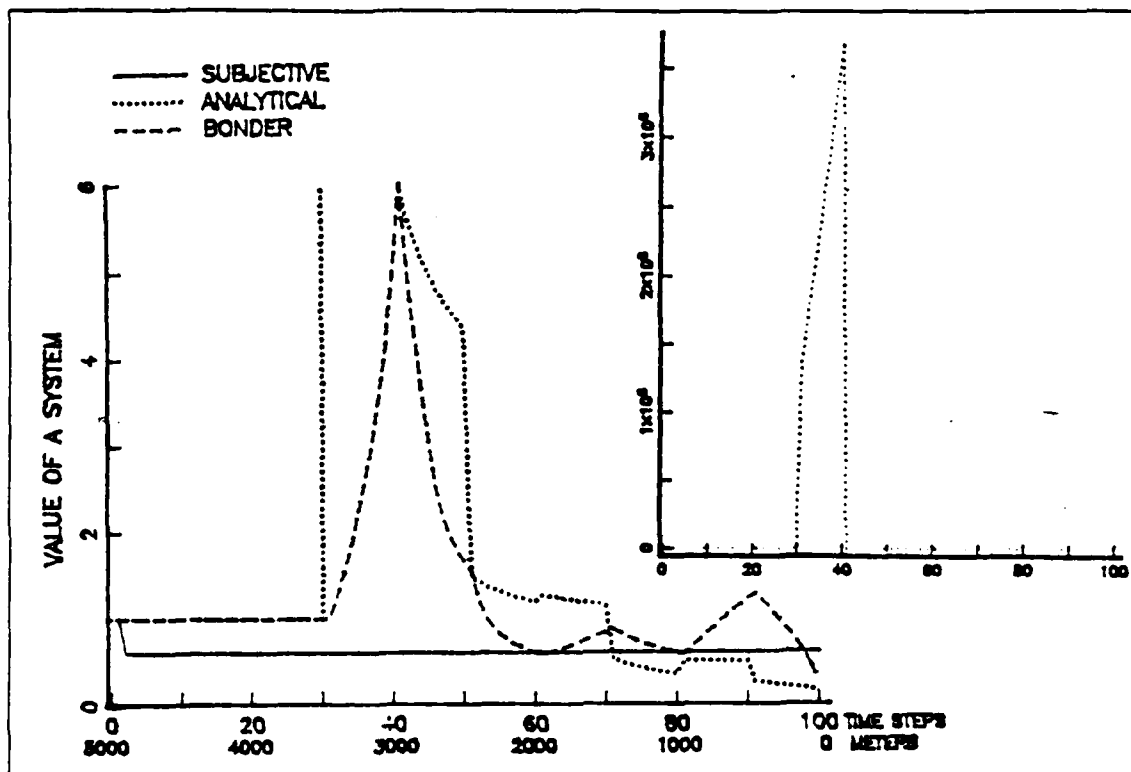


Figure 4.6 Potential-Antipotential Model - M2 IFV System Values.

d. Indirect Fire Systems - M109A2 and M110A2 SP Howitzers

The portrayal of artillery systems in the scenario assigns various weapon types constant killing rates, i.e. lethality is the same over the entire 5000 meter battle area. Since the artillery systems lethality and accuracy are not changed, initial expectation is that there will be little information for post simulation analysis. This assumption does not hold true. The information provided by the artillery output continues to support some but not all of the trends noted earlier for the various methodologies.

In the Lanchester model (Figure 4.7), the subjective approach results in the lowest number of combat losses and continues to exhibit a linear attrition rate. This is consistent with expectations considering the fact that the only system that can kill the M109A2 is the 152mm howitzer which has a constant attrition coefficient like all other artillery pieces. Thus the results reflect the conditions that they should mathematically follow. While the simulation result is consistent with the inputs, it is important to note that the portrayal of the artillery battle as being linear is not realistic. Further, this

linearity is a direct result of using the subjective approach. This suggests that artillery should be handled as a separate subroutine within the modelling process and not treated as another direct fire system with different parameters. This separation of indirect and direct fire system attrition development is incorporated by Bonder in his aggregated models. The analytical and Bonder data produce linear attrition flows as expected when the system lethality is constant throughout the simulation. The Bonder attrition rate exceeded those of the analytical and subjective approaches. This pattern repeats itself in the case of the M110A2 SP howitzer.

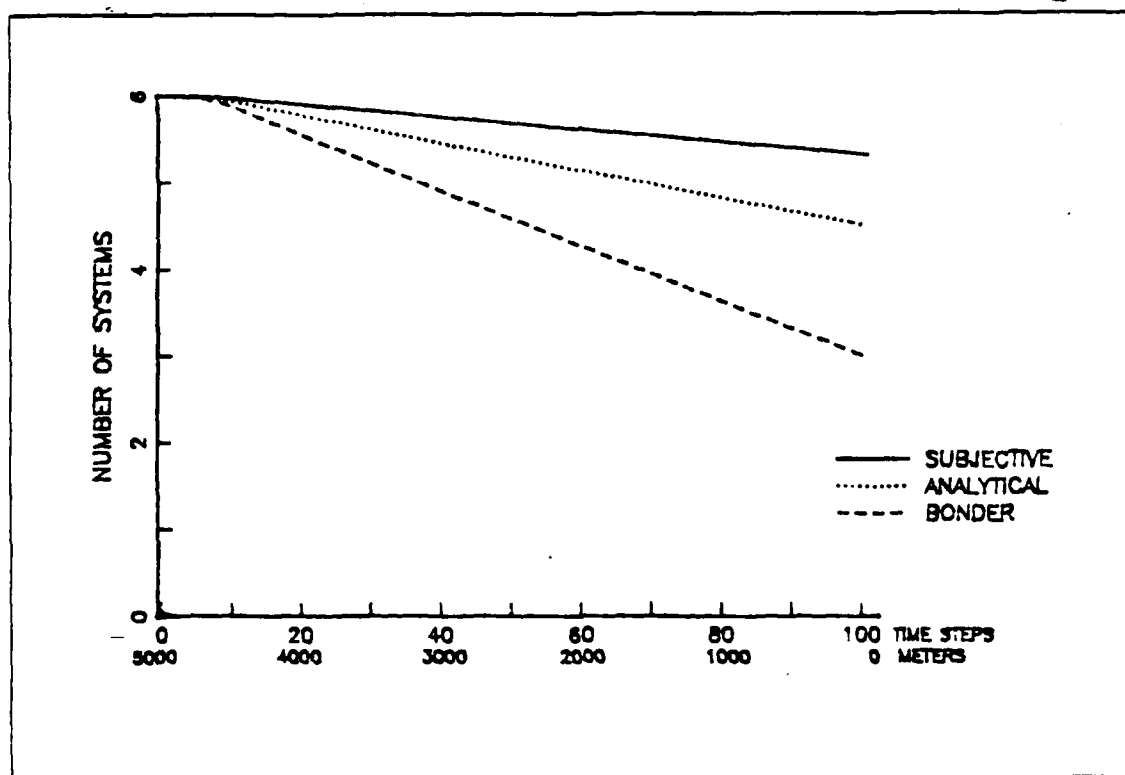


Figure 4.7 Lanchester Model - M109A2 Howitzer Combat Flow.

The P-AP model produces output with greater information content for the user than the Lanchester model. In the Lanchester model output, the affect of artillery on opposing system attrition in most cases can only be isolated from the effect of other systems in the indirect fire zone. In the P-AP model, the various affects of artillery are portrayed in the relative values observed throughout the simulation. While the subjective values can be generally ignored, the analytical output data displays several

aspects not specifically noted in other systems. First, the direct fire system values have tended to be greater than their Bonder counterparts across the simulation, this is not the case for the artillery (see Figure 4.8). The analytical values are greater at the longer ranges, as expected but drop below the baseline 1.0 value at the 3000 meter firer-target distance. Next, the radical increases and decreases observed with the ITV and M2 hinders the opportunity to conduct a reasonable analysis of the system values. Values range between 9.7 and 0.3 by simulation end. On either side of the model anomaly, the value appeared fluctuate within a small range of values. Analysis of this information reveals that as new systems entered the active arena, the value of artillery quickly lost value.

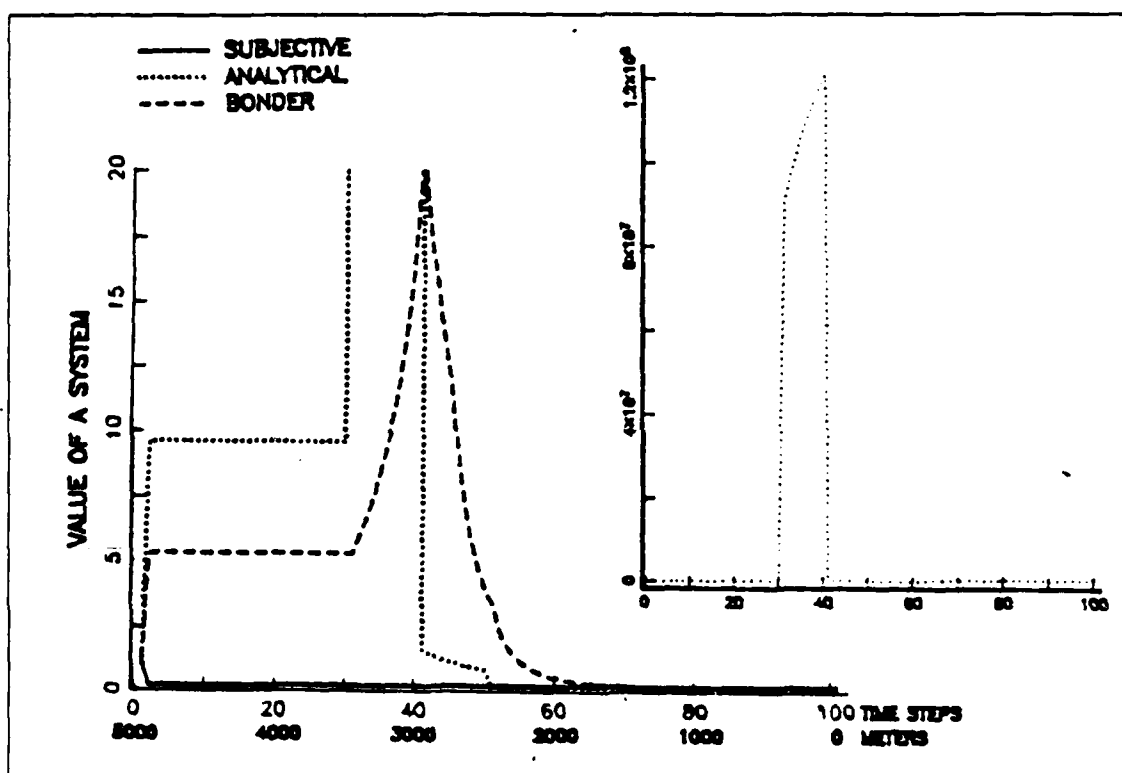


Figure 4.8 Potential-Antipotential Model - M109A2 Howitzer.

The Bonder approach produced system values between 0.1 and 20.0. However, initial evaluation of these results indicate they do not fit the tactical expectations normally attributed to artillery systems. The indirect fire zone shows artillery as being approximately 5 times the value of the baseline M1 system. This is

consistent with the military view of the battlefield; since artillery is the only system capable of engaging and destroying targets within this specific scenario. The unexpected increase in the system value, reaching a point approximately 20 times the baseline system at a range where the ATGMs are active killers in the system is difficult to accept at first glance. Subsequent analysis reveals this shift in amplitude corresponds with the points of maximum observed values of the opposing Red force weapon systems, thus supporting the apparently high relative values shown in Figure 4.8. As the killing potential of the tanks and ATGMs begins to increase, the constant lethality of the artillery also reflects the opposing system's increased values. Since a tank has more value as time progresses, the artillery piece gains value since it can now kill a system with a higher value. This effect is then accumulated across all systems and the end product is a steep increase in the system value of the M109A2 (155) howitzer. This trend quickly reverses itself at the 3000 meter point, when the tank system lethality surpasses that of the artillery. Review of the $P(k|h)$ matrices in Appendix E indicate that the 3000 meter matrix (interpolation of values) is the approximate point where tank lethality surpasses that of the artillery. Consequently, the behavior of the system values though surprising in their magnitude are consistent with the model assumptions and input data.

Similar trends were observed with the M110A2 (203) howitzer system for both the Lanchester and P-AP models. The only exception was the exceedingly high values produced by the analytical and Bonder simulation runs in the 3000-5000 meter range envelop. This behavior is attributed to the counterbattery mission that exists although the magnitudes in excess of 10^7 become immaterial for analytical purposes. These high value further suggest that adjustment within the model in how artillery is portrayed will definitely effect the outcome regardless of the closeness of input values.

e. Offensive (Blue) Force Summary

Analysis of the simulation outputs reveals several emerging patterns. Use of the Bonder equation generation methodology generally produces results consistent with the input parameters and model process. With the exception of the M1 tank system, the attrition flow, weapon lethality and vulnerability evaluation levels were consistent. This allows the client /user the opportunity access to additional information without extensive analysis or modification. Additionally, the Bonder results lie predominantly between the subjective and analytical output data points, suggesting that the latter methods may be under- or overestimators.

To conclude that the subjective and analytical approaches produce only under/overestimates of combat losses and system values is unreasonable considering the limited sample population surveyed. The conjecture will be seen to be false in the analysis of the Red force systems deployed in the defense. However, based on the available information, investigation of the estimation hypothesis relative to only offensive operations may be a topic worth pursuing in some future study.

The subjective and analytical approaches fail to provide the consistency of output shown by the Bonder technique. The subjective output in Lanchester is difficult to assess as the constant attrition coefficients produce only linear combat losses and are difficult if not impossible to correlate with the P-AP results. Thus the overall benefit in using this approach is limited at best. The analytical method, which reflects a more acceptable portrayal of combat, still results in areas of discontinuity and precludes a full examination of the cause and effect relationships of the simulated combat. However, the approach remains for most situations a viable alternative and with limited exceptions tracks consistently over the two models.

f. T72 Tank

In the previous sections the offensive weapon systems have been examined and several possible trends in the model outputs have been noted. The T72 provides the first look at the defensive forces as well as the first and only complete tank system analysis for both models.

The Lanchester outputs reflect a complete change from the offensive M1 tank profiles (see Figure 4.9). The subjective approach indicates a nearly constant attrition rate throughout the battle as expected. However, in this instance, it reflects higher rates of combat losses in the early stages of battle while the analytical and Bonder outputs reveal higher rates occur after the introduction of opposing direct fire systems into the battle. Interpretation of the constant slope of the subjective output suggests to the user that tank losses will occur at the specified rate regardless of the introduction of new systems such as the M1 or ITV. If the user is the offensive force commander, this may be construed as indicating that since opposing tanks can be killed at the same rate anywhere in the battle, it is necessary to only use artillery with its stand-off capability to win the battle. This is unquestionably erroneous because artillery duels and barrages in past wars, while inflicting enormous personnel and property damage, have not won the battles by themselves.

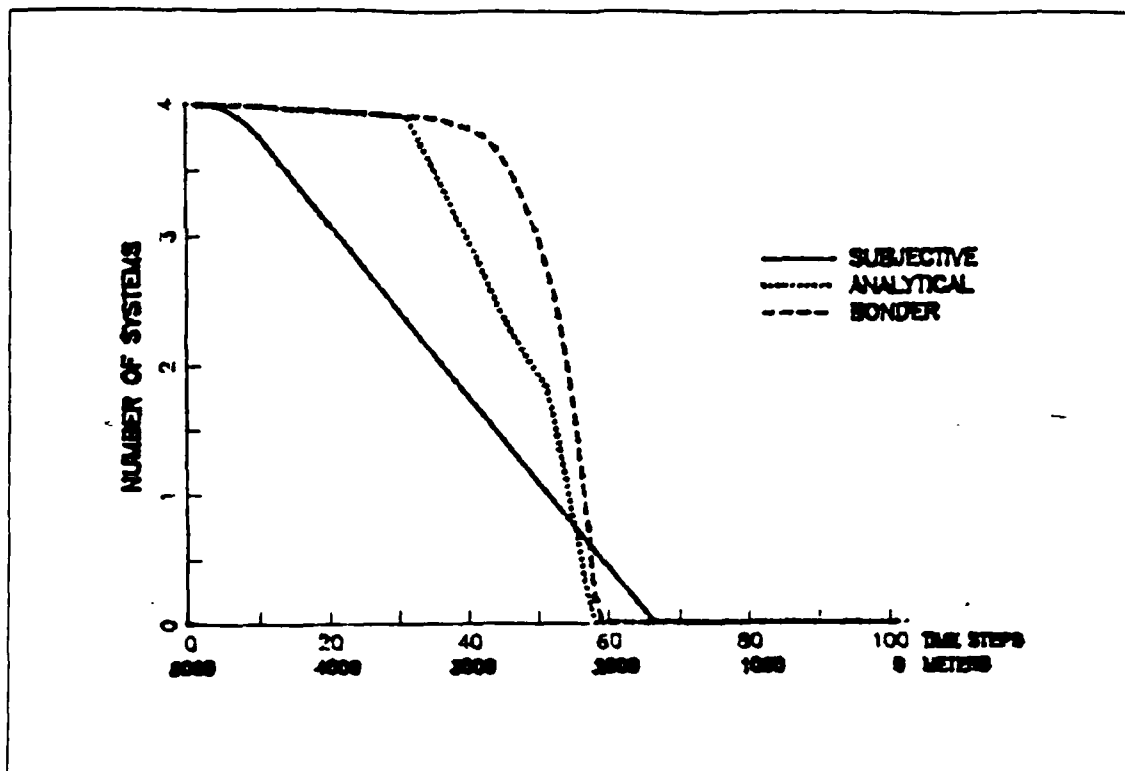


Figure 4.9 Lanchester Model - T72 Tank Combat Flow.

The analytical method produced results that were between the subjective and Bonder methodologies. Analysis of the combat loss line indicates two critical points where the system attrition rates significantly increase. The first point corresponds to the introduction of the TOW weapon system. The difference in the slope of the two line segments indicates there is a major difference in the expected losses when only artillery fires can engage the vehicles and when the AT guided missiles are used. The second point is near the 2400 meter firer-target range point. This equates with the M1 tank moving toward dominance on the battlefield. Both of these instances correlate with the normal expectations and intuitive reasoning.

The Bonder approach output portrayed the lowest attrition rate from 5000-2900 meters. From this point on the results reflect the highest rates of all three alternatives. While the subjective technique exhibited no changes as systems entered the battle, the Bonder approach reflected changes but could not be directly linked to the introduction of any one system. The largest changes occur near the ITV and M1 entry points but do not provide a distinct breakpoint for analysis. However, the fact

that the plot corresponds with activation of the direct fire systems into a firer role provides more information than the subjective approach but less specific data than the analytical method.

Initial examination of the P-AP output plots in Figure 4.10 provide an interesting counterpoint to the Lanchester model results. The subjective methods relative value of .95 provides a reference line near 1.0 designated for the M1 baseline. Both the Bonder and analytical approaches display values under this level until the 7.5 minute (time step 30) point of the simulation then portray two distinctly different relative-value profiles.

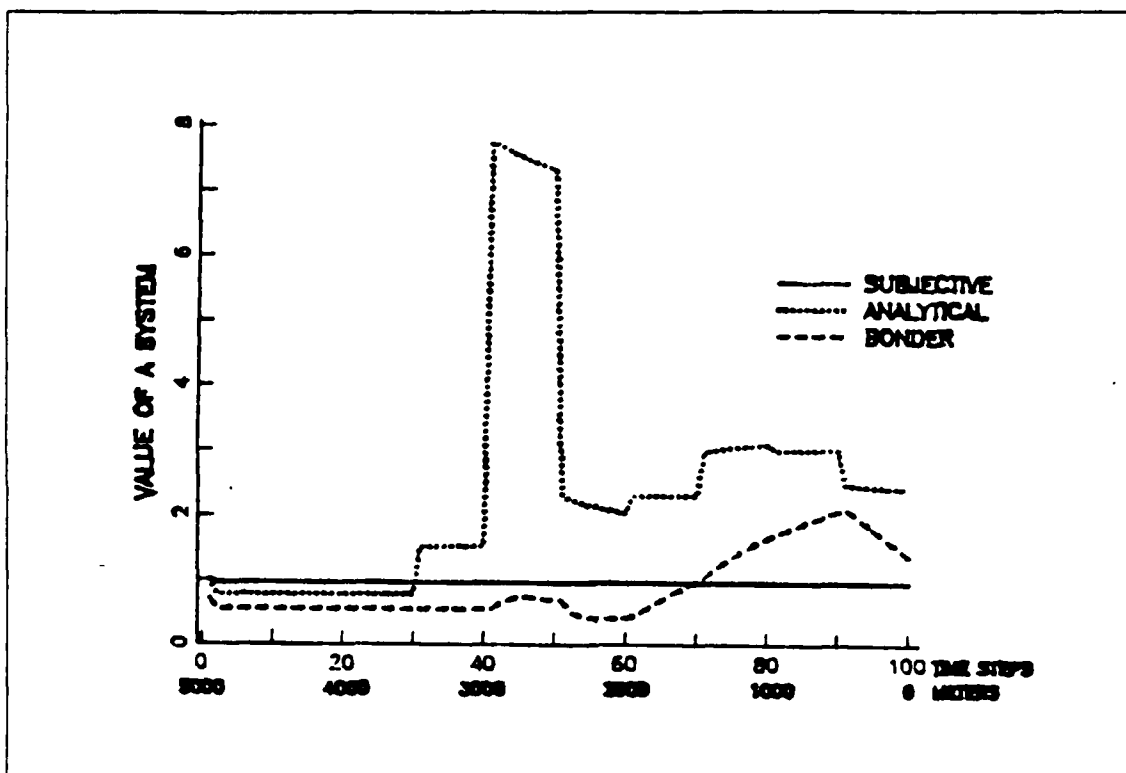


Figure 4.10 Potential-Antipotential Model - T72 System Value.

The analytical profile moves rapidly upward from the 3500 meter point, achieving a maximum value of approximately 7.8 in the 2500-3000 meter combat window. It then decreases to approximately 2.1 and remains between 2.0 and 3.0 for the duration of the simulation. This indicates that the T72 tank is of maximum value to the defender at a range beyond the main gun's maximum effective range. Further, it

suggests that the T72 is 2 to 3 times more valuable than the M1 tank. Initial reaction to this is to consider the method erroneous and ignore the results. This reaction can be countered by remembering that the T72 is defiladed and the M1 tank and other systems are in the open and moving. Thus if one applies the combat multiplier of 2.0-3.0 to the defender true value, these numbers fit acceptable limits for the 0-2500 meter firer-target window.

While the end values are explained by the tactical nature of the scenario, it should not be necessary to apply after the fact modifications to the results to analyze the systems. Further, explanation of the midpoint values are not as readily acceptable. A system value lower than 0.95 will result from the difference in the input variables used by the three methods. The constant value denoted by the graph in Figure 4.10 balances with the fact that only artillery systems are active during this phase of the simulation. The initial increase above the 1.0 level corresponds with the ITV system value increases but the magnitude of the succeeding increase does not correlate to the small values exhibited by the M1 tank as it enters into the firing battle. As previously stated, this is not intuitively acceptable and contradicts the nature of combat by allowing a system to achieve its maximum value before it can participate in the battle. An extreme translation of this concept would be to leave all the tanks in reserve; since they are of more value to the force commander than when they are actively engaged in combat.

The Bonder results run closer to the intuitive perception of the opposing systems contribution to the battle. Initial assessment of the T72 system considered the tank to be slightly lower in value than the M1 based on overall capabilities. This fact is reflected in the system values below 1.0 during the artillery battle when the tank is passive (i.e. only moving but not engaging targets). As the AT systems enter the battle, the tank, still passive, does not show an increase in its relative worth. A small increase in value occurs at the point where tank engagements may be initiated but are of minimal benefit due to the low probability of hit at extreme ranges. These values decrease again as the AT systems $P(\text{kill})$ become almost constant and the M1 tank's longer range and accuracy reduces the expected value displayed by the opposing T72. An increase in value begins as differences in the tank systems accuracy and $p(k|h)$ diminish until the T72's defensive advantage and greater round lethality outstrip the range and accuracy earlier available to the M1. A final increase in value correlates with the initial use of the 25mm cannon and decreases as that system's killing potential increases with decreasing range.

Review of the P-AP output strongly suggests that the Bonder attrition coefficient generation technique provides a more consistent and acceptable output than either the subjective or analytical FPS techniques. The discontinuity of the analytical approach's result near the midpoint of the battle significantly detracts from the credibility of the values for both the early and late stages of battle. Likewise, the need to apply combat multipliers to final output to bring values into alignment with the other methods further reduces the acceptability of the associated output values. The subjective results fail to reflect the changing nature of the battlefield and provided the user no more information after the P-AP simulation than before the start of the model runs.

g. AT5 ATGM

The Lanchester model results produced the same general patterns observed for the T72 tank. However, output from the P-AP simulation did not follow the same trend but did correlate closely with the output analyzed for the ITV TOW system.

The analytical and Bonder results appear as variable scaled versions of their Blue force counterpart when examined within the three basic fire zones. This is understandable when comparing two systems with nearly identical weapon characteristics. The differences observed for the AT5 occur at either end of the simulation. At the longest ranges, the artillery/indirect fire zone, the AT5 takes on a value which is less than 1.0. This is unsettling when considering the almost identical system on the opposing force carries a 1.0 evaluated weight. This is also the case for the Bonder output data. (See Figure 4.11)

As with the TOW system, the AT5 value takes on values with magnitude of 10^6 , far from a usable scale in the 3000-3500 range window, then decreases in corresponding steps to simulation end. The critical difference is that the assigned weight of the AT5 at the 2500-1500 meter firer-target ranges reflects a value over twice that of the TOW system. In this instance, the differences in opposing systems values does not support the expectation of a system weight over twice that of the Blue force system. This discrepancy disappears by the 1500 meter separation mark as values track with the TOW. This also is true with the Bonder approach output with sub-1.0 values before the system enters active combat and the value of the system approximately twice that of the TOW in the 2000-3500 range window. In both cases, the Bonder approach provides the most credible and usable data.

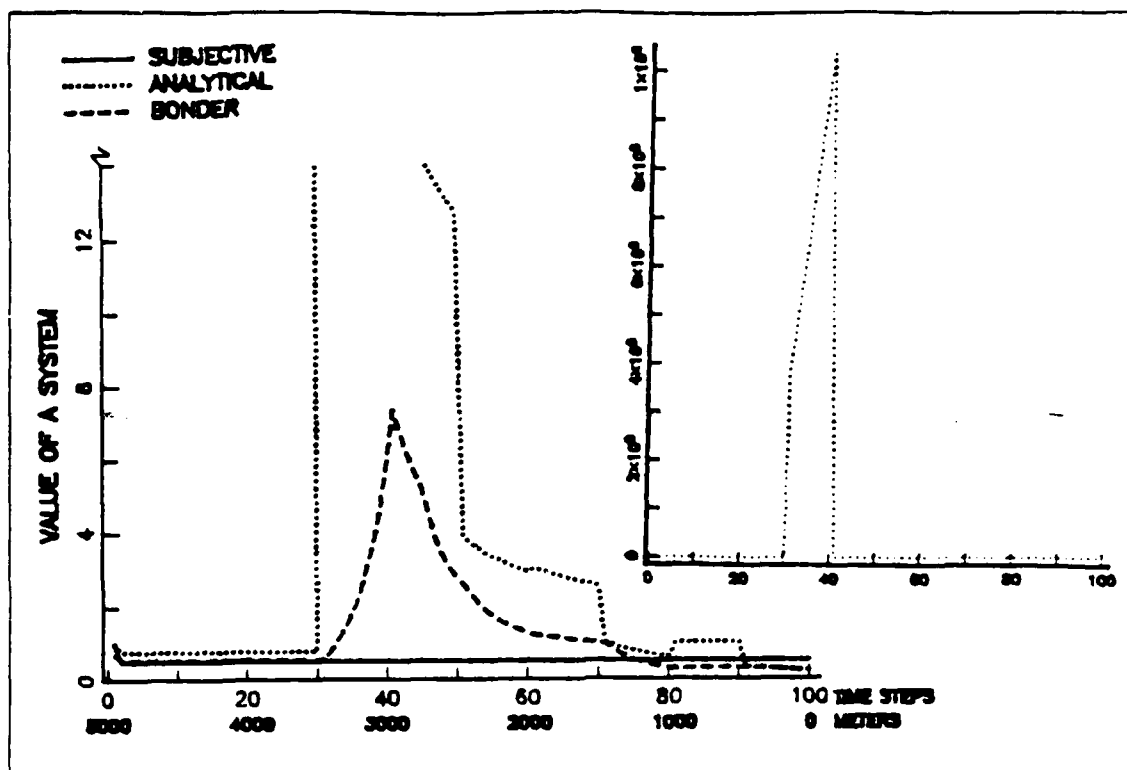


Figure 4.11 Potential-Antipotential Model - AT5 ATGM System Values.

h. BMP IFV

As expected, due to identical opposing system entry points into the battle, a similar pattern for the subjective, analytical and Bonder attrition coefficient generation methodologies outputs resulted from the Lanchester model simulation. The constant rate of the subjective approach was highest for the indirect fire battle and the first half of the long-range battle. After this point, the higher rates were found in the analytical and Bonder simulation results (see Figure 4.12).

It is interesting to note that in each subsequent system, the values for the analytical and Bonder results become closer to each other. This suggests that under certain circumstances in a Lanchester Square Law simulation the simpler analytical FPS approach for attrition coefficient generation may be a satisfactory substitute for the more detailed Bonder generated coefficients. In the case of the BMP, there is a point of intersection near the 55th time step (see Figure 4.12).

The P-AP system value results shown in Figure 4.13 create a more interesting analytical exercise. However, before analysis is undertaken, it is necessary

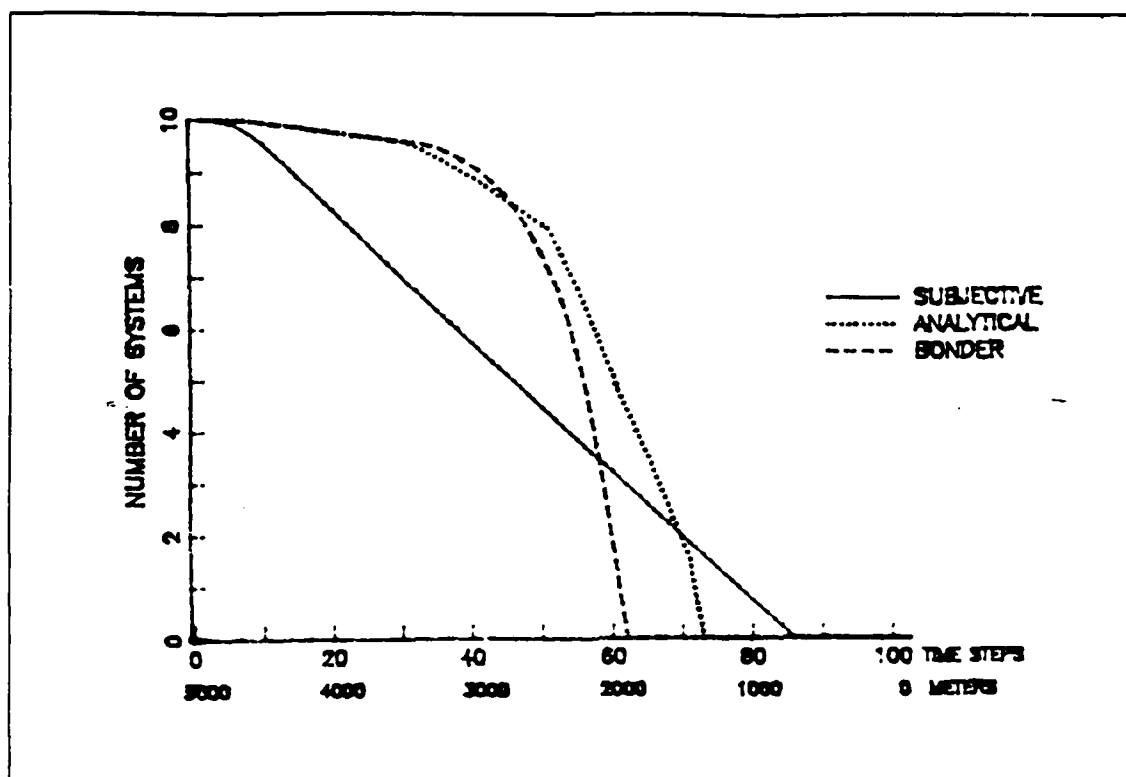


Figure 4.12 Lanchester Model - BMP IFV Combat Flow.

to clearly state that the BMP is considered for the purpose of this scenario as a single weapon platform with the 73mm gun and not a multi-weapon vehicle like the M2. The analytical technique and Bonder method produce distinctly different interpretations about the BMP platform. Initial evaluation of the system indicates a relative value of below 1.0. This is consistent with input data and consideration of its capabilities in the passive role as being less than a M1. However, at the 3500 meter point, the value increases to approximately 1.6, retains this value for 500 meters or 2.5 minutes of simulated combat, then decreases to near zero. The BMP's value continues to hover near zero until it reaches the firer-target distance of 2000 meters. At this stage its value begins to increase, stabilizes near 0.5 then moves rapidly to above the 1.0 mark again. It then unexpectedly drops to the 0.7 level and decreases slowly until simulation termination.

Explanation of these phenomenal value changes does not correlate with the currently accepted concept of the modern combat environment. The value of an active system at ranges beyond its engagement range are intuitively incorrect. Under the 'kill

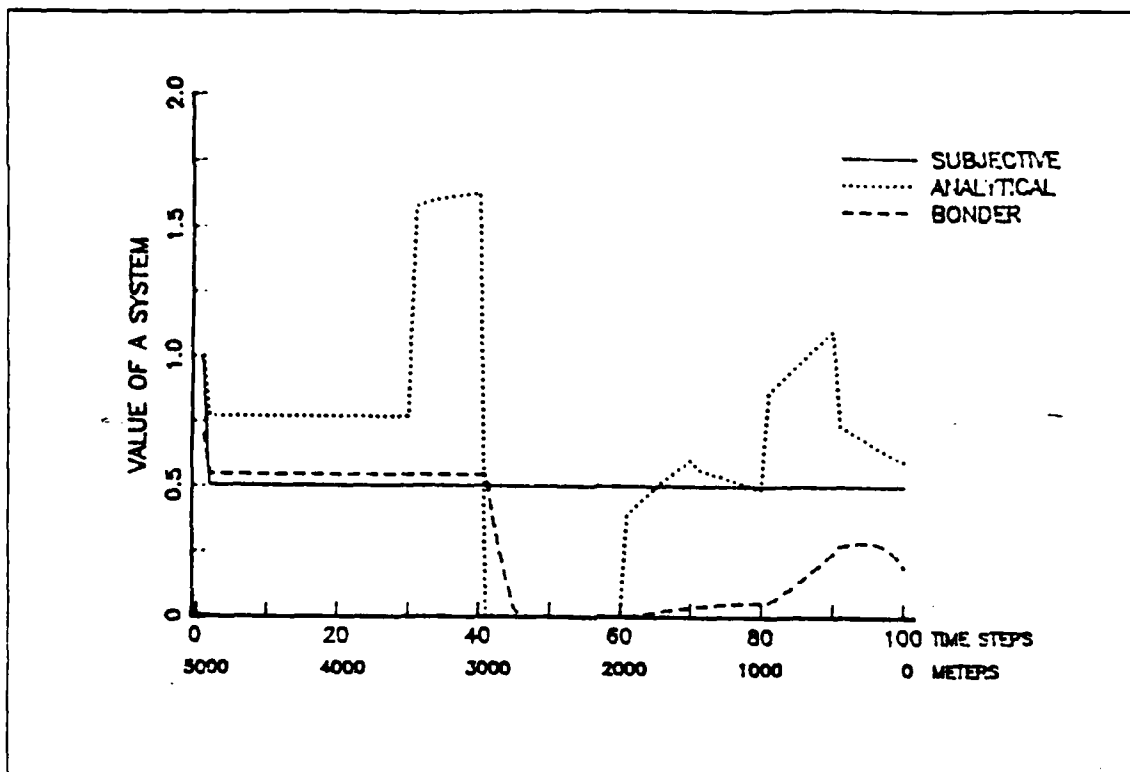


Figure 4.13 Potential-Antipotential Model - BMP System Value.

or be killed by' evaluation of the P-AP methodology, the large incremental movement upward is caused by the extremely large values observed for the other weapons systems at similar ranges of the analytical output. These overinflated values for the Blue TOW, M2 and artillery systems therefore created a false image of the BMP's contribution to the Red defensive force. Such an incorrect representation could lead the force modernizer to consider the need for additional systems to counter the BMP when sufficient systems already exist within the military inventory.

Once the values for Blue force systems have returned to levels commensurate to their role in the combat scenario, the BMP's relative value drops to zero. This low value is acceptable considering the passive role that the BMP plays at this point. Once inside the 2000 meter range to target mark, system value increases follow a trend consistent with increased weapon effectiveness for the 73mm gun and opposing weapon system. The decreased in system values observed at the 500 meter mark relate directly to the rapid improvement in the M1's lethality inside the 1000 meter firer-target point.

i. Indirect Fire Systems (122mm and 152mm SP Howitzers)

The results produced for both the 122mm and 152mm SP artillery systems provided no additional insights or significant deviation in the trends previously observed in the other weapon systems. The subjective FPS technique produced simulation results reflecting the lowest attrition rates, followed by the analytical FPS and Bonder methods, respectively. This pattern held true for both the 122 and 152 systems. Likewise for both systems, the analytical and Bonder approaches produced outputs that were close in value. This was expected since the methods used to model artillery varied little between the two approaches.

In the P-AP simulation, the analytical technique output continued to experience excessively high values in the 3000-3500 meter window. Also the 152mm system exhibited extremely high values until after the 3000 meter point. The artificial value used to represent zero for the purpose of matrix operations is responsible for the high values. Therefore, modeling considerations prevent a detailed analysis of artillery values at the long and middle ranges.

C. SUMMARY

The three methodologies have produced a variety of output results, but still do not provide sufficient data to categorize their effects on model output by any single rule. Further, because the end product takes on different meanings for the various users, there is no single answer as to which method is the best approach to follow. However, review of the weapon system analyses indicates several fundamental patterns in the Lanchester and Potential-Antipotential results.

In the Lanchester model outputs the subjective FPS technique produced results of little or no informational value to the user. The consistently linear results failed to reflect the changes in target profiles expected when new firer systems entered the battle thereby enhancing any estimates in favor of long range weapons. For the offensive force systems, the subjective approach output chiefly produced a conservative estimate. For the defensive forces, the technique overestimated losses at the longer ranges and underestimated attrition in the close-in battle zone. Finally, the majority of results were deemed unacceptable under the given scenario conditions.

The subjective FPS method provided output of minimum value in the P-AP model unlike the Bonder or analytical FPS techniques. Since the output of the P-AP model measures the relative worth of the weapon system, the output is no different

from the input. Therefore, use of the subjective method in a P-AP model provides the user with unrealistic and uninterpretable information. While the values generated in the model at times appear realistic, the majority of values do not approximate the value of a system which is known to be effected by various combat conditions. For all the systems and model types, the subjective approach provides the least useful and realistic representation of the combat process.

The analytical FPS technique provided far more information to the user than the subjective FPS. In the Lanchester model, the results were believable and reflected the introduction of new firer platforms by clearly identifiable points on the attrition plots. Consistency as an under- or overestimator was not noticeable across the spectrum of available weapon systems. However, in the case of the

unit, the approach tended to estimate higher combat losses for the direct fire system than the other two approaches.

The P-AP model outputs for the analytical method did not provide as consistent and realistic interpretation of system values as the Bonder approach. It appears that the analytical generation approach may be more sensitive to external model influences than the other techniques. Consequently, the tendency for a system to display a relative value that is intuitively impossible on an actual battlefield significantly detracts from the desirability of this approach for attrition coefficient generation. While it was previously noted in Chapter II that a more sophisticated model would be able to avoid these values, it is an external modification to the actual generation methodology and becomes a modelling question and not an attrition coefficient generation topic. Further review indicates that the analytical approach consistently produced the highest values in each of the three combat fire zones suggesting the possibility of it being considered an overestimator.

Attrition coefficients generated directly by the Bonder equation and run in the Lanchester Square Law simulation model provided with few exceptions the most consistent and believable results. Initial trend analysis indicates that the model outputs adhere to the hypothesis that as ranges decrease attrition rates should increase. The analytical approach provided distinct firer system entry points while the Bonder results reflect a smoother transition across the combat simulation. Whether this is a more accurate portrayal of the sometime swift and violent nature of combat is left to be decided upon by those people with first hand experience and data.

The outputs produced from the Bonder coefficients in the P-AP model again provided logical and consistently acceptable system values which were equally usable by the various clients of aggregated simulation runs. The data produced showed little or no effect from model constraint influences and provided reasonable data throughout all three combat fire zones. Overall, the Bonder equation outputs were analytically sound for either model. Further, these results were consistent between the Lanchester and P-AP models where the subjective and analytical results were not markedly consistent.

V. SENSITIVITY ANALYSIS

The previous chapters have been concerned with the effect each attrition coefficient generation methodology has had on the pace of battle, attrition flow, and the relative weapon values throughout the simulation process. Included in this analysis was an examination of each technique's ability to accurately portray combat based on a given scenario. In all cases, the analysis was based on a fixed force size and a single set of weapon parameters. This chapter examines the numerical sensitivity of the methodologies and the effect on the model output interpretation as it pertains to force mix options.

The question of optimal force mix considers two specific aspects. The first is how many of each weapon type is required to achieve the mission. The second is how system replacement or improvement will influence mission accomplishment or resource allocation. In order to investigate the effect of changing input parameters, it was decided to make specific changes to the weapon characteristics of a single system, the 25mm cannon on the M2 IFV. Weapon characteristics were hypothetically increased by the use of hypervelocity round with over twice the current round's muzzle velocity. Additional increases in the systems $P(k|h)$, $P(h|h)$, and firing cycle times were included in an attempt to reflect the effects that such a round would exhibit when compared to the original round. A tactical modification accompanied the change which allowed the 25mm gun to start engaging targets within the 2000-2500 meter range window, thus resulting in the M2 shifting from the use of the TOW missile to the 25mm cannon approximately 1500 meters earlier than in the original scenario.

The outputs from the Lanchester and Potential-Antipotential methods are examined from two perspectives. The first considers the effect of the introduction of the new weapon on the outcome of the battle. The second aspect is directed toward the force mix and what trade-offs in the force structure could be made to achieve the same battle outcome. Analysis of the latter aspect will provide the mission dependent weapon trade-off values not considered in the baseline scenario. Finally, sensitivity to force size changes will be investigated using the baseline attrition coefficients to determine if increased force size has any effect on the simulation output.

A. SUBJECTIVE FPS

Introduction of the new weapon characteristics into the analytical and Bonder methodologies requires changing clearly defined mathematical inputs. Determination of a new subjective value for the M2 based on weapon changes is an entirely different process. However, for the purpose of the sensitivity analysis it was assumed that the new M2 subjective value would be approximately 1.28 times that of the M1. This value was based on the changes in the observed values generated by the Bonder equation. The new relative worth and the associated B_{ji} attrition are shown in Table 16.

TABLE 16
IMPROVED 25MM GUN SYSTEM

Relative Worth Matrix					
	T72	ITV	BMP	122	152
M1	1.0526	2.0000	2.0000	6.6667	6.6667
ITV	0.5300	1.0000	1.0000	3.3333	3.3333
M2	1.3474	2.5600	2.5600	8.5333	8.5333
155	0.1578	0.3000	0.2000	1.0000	1.0000
203	0.1578	0.3000	0.2000	1.0000	1.0000

B_{ji} Attrition Coefficients					
	M1	ITV	M2	155	203
T72	0.0105	0.0530	0.0134	0.0015	0.0015
AT5	0.0200	0.0100	0.0256	0.0030	0.0030
BMP	0.0200	0.0100	0.0256	0.0030	0.0030
122	0.0662	0.0333	0.0853	0.0100	0.0100
152	0.0662	0.0333	0.0853	0.0100	0.0100

Introduction of the improved 25mm gun into the Subjective FPS model produced a decrease in the total time for combat mission completion by the Blue force. The decrease represented 1.5 minutes or a vehicle closure distance 300 meters less than the baseline simulation. Since only one system was changed, the improvement in the battle outcome can be attributed solely to the 25mm gun upgrade. Analysis of input data using standard FPI and force ratio equations reveals that the firepower index for the Blue force increased to 56.18 or a 7.8 percent increase in value. The associated force

ratio improved to 3.601 (+7.8 percent) and the time for overall combat until mission completion dropped to 20 minutes for a 7.5 percent change. Evaluation of the data indicates that changes to the subjective inputs result in linear changes throughout the simulation. Since the subjective FPS is an interval scale, any linear transformation on scale will produce another interval scale without loss of information. Consequently, any increase or decrease in either force's FPI should be reflected proportionally across the entire battle simulation and provide predictable results without the need of extensive simulation. Further, examination of the results of changes in the P-AP will, as before, provide a minimum of new information to the user.

Regarding the force trade-off relationship, the prediction that the linearity of the system's relative value is retained appears to be correct. Initial comparison of the two 25mm M2 IFV systems indicates an expected exchange rate of 2.133; that is, for each new M2 system added to the force structure 2.133 old systems can be removed. Likewise, trade-off rates of 2.56 for ITVs and 1.28 for M1 tanks are projected based on the subjective firepower ratios. Various force composition combinations were tested with the criteria that mission duration endpoints be approximately 21.5 minutes (± 15 seconds). Regression analysis on the force composition inputs produced the results presented in Table 17. Comparison with the predicted exchange rates and the regression analysis rates produces nearly identical results with the variance accounted for by rounding errors. The assumption that the linearity in system values is retained is supported by the correlation coefficients associated with the regression analysis.

TABLE 17
REGRESSION ANALYSIS OF WEAPON SYSTEM TRADE-OFFS

System	Regression Eqn	Correlation Coefficient
M1	$Y = 45.15 - 1.13X$	-0.9929
M2	$Y = 2.0X$	-1.0000
ITV	$Y = 17.2 - 2.5X$	-0.9975
Y = number of systems X = number of improved M2 systems		

Using the subjective FPS values in the P-AP model produces significantly different results than was expected after reviewing the Lanchester Square Law outputs. Since the P-AP values are determined by the relationship of the firer-target pairs, the Red force system values, rather than decreasing in relative weight, actually increase.

Excluding the planned increase for the M2 IFV, the Blue force system scores remain unchanged except for the 203mm SP howitzer which decreased by 5.5 percent. This decrease is inconsistent with the P-AP model premise that a system's value is relative to those systems that it can kill or be killed by opposing force systems. Red force output indicates that 4 of 5 systems exhibited 17 to 21 percent increases in their value. The single exception was the 152mm SP howitzer which displayed a 1.2 percent decrease. This unexpected decrease for both the counterbattery artillery systems supports the previous observation that artillery systems should be handled in a separate-subroutine from the direct fire systems if realistic outputs are to be obtained. Further, evaluation of the force exchange relationships reveals that the linear transformation effect permits an infinite increase in system numbers without any degradation of effectiveness. This allows the battle to be won at time step 1 whenever a sufficiently high force ratio exists even when forces are outside of a system's capabilities to kill targets. This violates the of the law of diminishing returns. While the subjective approach produces proportional and linear changes across all systems in the Lanchester Square Law model when a change is made, it also leads to an interpretation that corresponds closely to the Lanchester Linear Law that suggests that the side with the largest force value will win the battle. Thus while the method provides for easily interpretable system trade-off relationships, the approach fails to recognize any diminishing return as force sizes increase beyond the saturation point. Consequently the user may be mislead into thinking that a sufficient quantity of a system is a viable alternative to a force mix of several systems.

B. ANALYTICAL FPS

In order to portray the introduction of the improved 25mm gun it was necessary to upgrade the $P(k|h)$ values used for the baseline M2 system and generate a new series of attrition coefficients. Force composition was then modified to create new mixes that would produce measurable changes in either battle duration or weapon count requirements. To this end, several different results were noted that indicate that changes in analytical FPS attrition models do not adhere to the strict linearity observed in the subjective FPS results. The following is a listing of the major force exchange relationships observed from the model outputs:

- the presence of the new 25mm gun allows an immediate reduction of three tanks without effect on battle outcome

- a force with seven improved M2s allows the number of tanks to be reduced to 30 (an additional eight tanks)
- a force of eight improved M2s allows all tanks to be removed from the force
- six improved M2s allow the number of ITVs to be reduced by six to a total of four
- five improved M2s are needed to replace the six baseline M2 systems.

Since no proportional linear relationships in the attrition flows or system values was observed in the analytical FPS simulation results, it was necessary to examine sensitivity only in terms of deviation from the previous attrition level and system values. Overall Blue force system outputs from the Lanchester model simulation remained unchanged. This correlates with the fact that the Red force attrition coefficient matrices remained unchanged from the original model runs. Initial Red force attrition losses duplicate the rates developed for the original system in the indirect fire and the first 1 kilometer of the long-range fire zones. At the 3000 meter point, the Red force attrition rates increase as the improved 25mm gun system replaces the TOW as the primary weapon. This follows expected behavioral patterns since the increased killing potential of the M2 affects all the opposing systems. Similar deviations from previous loss rates were observed in the T72, AT5, and BMP loss curves.

The sensitivity of the analytical FPS based Lanchester models to change results in a more detailed output than the subjective FPS methodology. The effects of the weapon changes such as the increased range and lethality is reflected at specific points of combat and not averaged across the entire battle. This prevents overestimation of the effect of the weapon improvements in the early battle. Specific details about the impact of weapon performance can be observed in the P-AP model outputs which provide insight into the possible synergistic effects of weapon upgrades (see Appendix G).

Continuing to use a relative base of 1.0 for the M1 tank, examination of the differences in the relative value between the improved M2 and the baseline version of the system indicates significant changes to the value of the improved system (see Figure 5.1). Overall evaluation of the improved system resulted in extremely large values as did the baseline system in the 2500-3000 meter range window. At the 3000 meter range point, the relative value of the upgraded system exhibited an increased value of 110 points. Consideration of this change in value with the actual projected values of the original system (3.6×10^8) produces a change of .000031 percent,

significantly small to be considered as no change. However, further examination of the new system values inside the 2500 meter point reveals value changes from 200-1100 percent. Interpretation of this new data output indicates that an improved M2 system at a particular point in the battle is worth 22 M1 tanks, an unrealistic overestimation of system values.

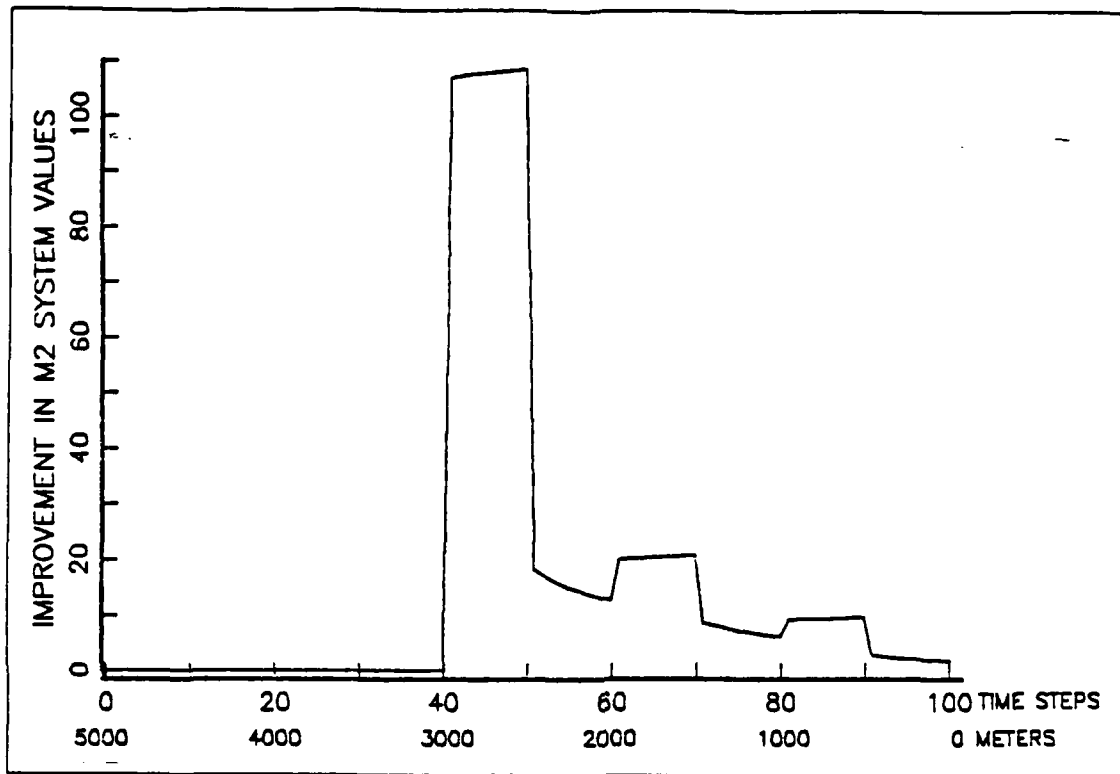


Figure 5.1 Comparison of M2 Systems In the P-AP Model.

The relative value of all Red force systems exhibited marked increases with the introduction of the improved 25mm gun. The T72 and the AT5 ATGM systems reflected maximum increases of 16 and 33 units, respectively. While these values are considered high, they reflect the large increase in value of the M2 system over the same time period. These values decrease to more acceptable levels, although still high, at the 2500 meter firer-target distance. From this point, value changes for all systems are between 0 and 4 points with the 152mm system displaying the least changes in value.

Closer examination of the output suggests that the analytical FPS output manifests itself in higher than acceptable values when fewer system interactions occur.

This is evident when viewing the artillery systems which exhibit the least change in system value while having maximum interaction with opposing systems. Analysis of the Blue force system values for the ITV and artillery systems, which are weighted against the new M2 system and revalued Red force weapons, show value changes in the ± 0.002 range. Further, the effects of changes in the analytical FPS approach are more precisely defined as to the time and location of occurrence than the subjective FPS results.

C. BONDER ATTRITION COEFFICIENT

Modification of input vectors and matrices to account for the characteristics and tactics associated with the improved 25mm gun system were simulated in both the Lanchester and Potential-Antipotential model environments. Overall combat duration was 15 seconds less than the baseline model indicating little or no change in the system relationships. Analysis of attrition data from the Lanchester model indicated that changes in the individual weapon system attrition rates were extremely small. This outcome was expected for the Blue force systems because the rates are dictated by the Red weapon system coefficients which remained unchanged. However, Red force loss rates were expected to be significantly larger than the observed attrition flow.

Examination of selected attrition matrices computed with the new input values indicate that the B_{ji} attrition coefficients values for the M2 against the T72, AT5 and BMP systems at 2100 meters differ from the baseline values by 6.34 to 11.8 percent, and at 700 meters between 18.1 percent and 296 percent. First impressions dictate that the output should then reflect these large coefficient changes as noticeable attrition rates. This was not the case. Upon closer investigation it is possible to determine that the small change to the combat loss profiles results from the magnitudes of the coefficients. Although the coefficients may double or even triple in value, a coefficient increase of 0.01 still requires the equivalent of 100 iterations to produce 1 additional casualty. Therefore the results obtained with the Bonder Equation methodology reflect minimal information about the actual amplitude of coefficient changes in a Lanchester simulation model. In cases where force sizes are small (battalion size) this tendency to provide minimal information about attrition flow changes will be more pronounced. However in large aggregated models (division level and above) observation of system losses will be more apparent (i.e. a larger number of systems will be killed) even though the rates will be the same.

The Potential-Antipotential model produced output consistent with expectations. Blue force values exhibited no noticeable deviation from the baseline values with the exception of the improved 25mm system. The improved 25mm system values were consistently higher than the baseline case inside the 3500 meter firer-target separation point, with a maximum value increase of less than 200 percent at any point with the range window. The Red force displayed a more balanced increase in system values against the baseline weights than in the analytical FPS technique. The observed output from the Bonder approach still provides the most believable, acceptable results.

The discrepancy between the change in system values and attrition rates present the user with two main areas of concern. First, while the system value changes are most apparent across the system which is upgraded and opposing force systems, any measurable improvement against the friendly system is almost non-existent at first glance. This precludes the force modernizer and force planner from making effective decisions without additional examination and analysis of the available data. Second, minimal changes in the Lanchester model output tend to indicate that major weapon improvements may have little or no significant effect on the battle outcome and therefore result in rejection of needed system upgrades.

In order to derive usable information for the product user, it becomes necessary to review and analyze multiple simulation runs to determine the pertinent data necessary for specific decisions. This is time consuming but essential if the decisions are to be made based on all available information. For example, initial graphical analysis indicated that there is little difference in the relative value of the M2 but sufficient to suggest some weapon trade-offs that could be considered. Simulation results indicated that the baseline 25mm force would require 18 M2s to replace 12 M1 tanks, while only 9 improved M2 systems would be required to replace the same 12 tanks. These same force relationships can be utilized to create additional weapon trade-off relationships. More importantly, the data analysis results in more realistic relationships from the military perspective and provides information that correlates favorably among all weapon system exchange rates. While the Bonder equation results may require a more indepth analysis prior to use by the decision maker, it does present a more acceptable result when compared with empirical data than the analytical approach.

D. FORCE RATIO ANALYSIS

The effect of changes in weapon system characteristics should produce a corresponding affect in the overall force ratio. Increases or decreases in the system values are expected to produce a change in the force ratio in a direction consistent with the value change. Therefore, an increase in the attacking force's firepower index, the sum of the product of relative weights and the number of systems available, should produce an increase in the force ratio. Analysis of force ratios produced by the three methodologies reveals that each ratio, while analytically correct, provides the output information that is intuitively incorrect to the military planner.

In the analysis of the subjective FPS force ratio, it is possible to derive two different values. The first, taken directly from the values found in Table 4 and Table 16, results in a force ratio of 3.601 (in increase of 7.8 percent). Computation of the ratio in the P-AP model produces a value of 2.979, a 6.5 percent decrease from the previous P-AP force ratio. This decrease in value contradicts the basic assumptions of the firepower scoring methodology. To consider a force with increased capabilities to have a smaller force ratio than before the improvement is intuitively incorrect. This result provides further evidence that the use of values derived from the subjective FPS approach for generation of attrition coefficients should not be used in the P-AP model environment.

Review of the individual changes in system values reveals that the overall unit FPI changes in favor of the threat force. This is based on improvements observed over the majority of the Red force systems while only the M2 systems in the Blue force exhibited any significant change. Thus the decrease in the force ratio, while consistent in terms of the P-AP model approach, contradicts the key assumptions behind the subjective firepower score methodology.

The analytical FPS force ratio with the improved 25mm system resulted in output that like the subjective FPS technique is initially contrary to military expectations. Without examination of the individual system value changes that occur when using the P-AP model, the force ratio plot (see Figure 5.2) is logically inconsistent. At the 3000 meter firer-target mark, the force ratio increases to approximately 1.25, indicating the ratio favors the attacker. This corresponds to the point where the improved 25mm system enters the active battle. As the simulation progresses, the force ratio continues to improve as the new M2 system and the M1 tank begin to dominate the battlefield. Throughout this period, the $P(k|h)$ of the M2 increases. At the 1400 meter point, the

force ratio falls below the old ratio which was based on the old 25mm gun. This contradicts the intuitive estimation process as the M2 continues to increase in lethality.

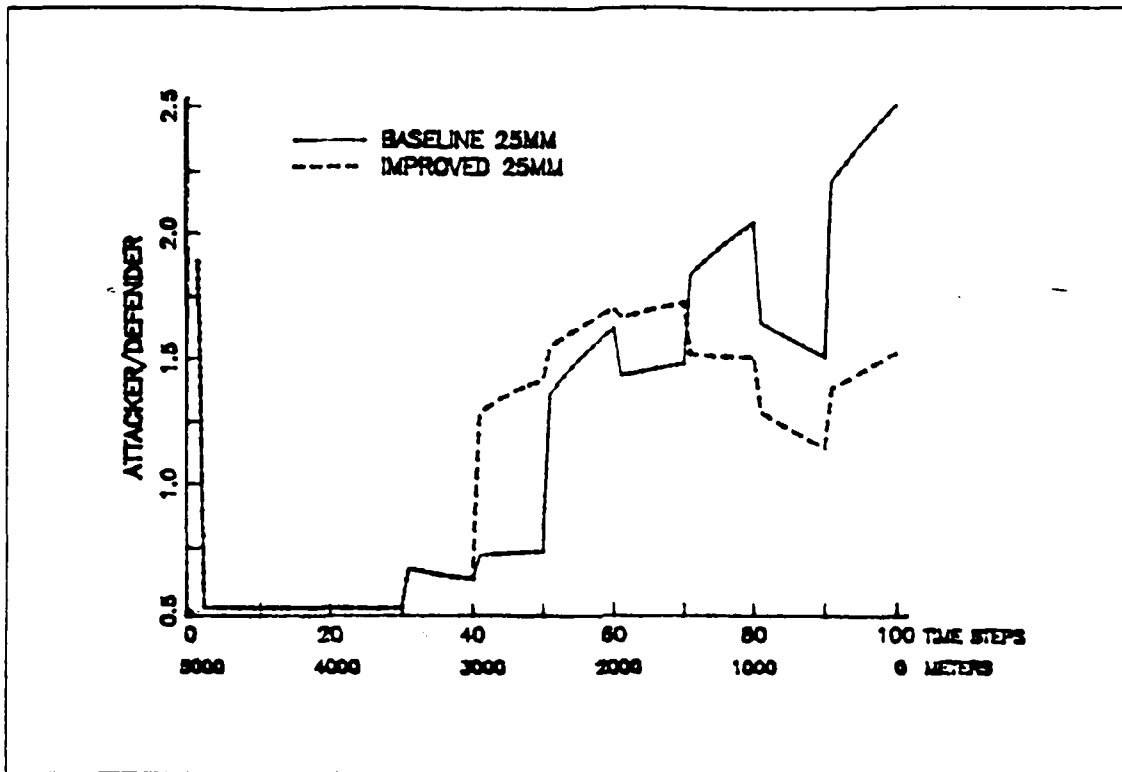


Figure 5.2 Comparison of Force Ratios - Analytical FPS Method.

Assessment of force ratios, in general, suggests that a monotonically increasing attrition coefficient for an attacking system should produce an increase in the force ratio throughout the simulation. Contrary to this assessment, a decrease in the force ratio was observed in the close-in battle zone. This observed decrease is caused by two factors. The first is that the increase in the M2's relative value creates subsequent increases in four of the threat force systems while for all practical purposes, none in the Blue force. Therefore the overall FPIs generated at each interval will at some point shift in favor of the Red forces. The second is caused by the calculation process used in the models. The force values which are determined throughout the simulation continue to exist even when the an opposing system is eliminated from combat. Therefore when the last M2 is killed the Red force systems are still calculating their value on their potential to kill the M2. Based on these observations, the analytical

FPS method's sensitivity to changes in system parameters produces results consistent with the P-AP model methodology but contrary to military expectations. Consequently, use of this approach requires additional interpretation before it can be effectively used by the military client.

When using the Bonder methodology the resulting force ratio exhibited the least change of the three approaches. Overall examination of the force ratio values indicate a decrease in the ratio with the introduction of the improved 25mm system. Both of these observations correspond to the increases noted in the Red force systems overcoming the increase seen in the M2 system weight and the minimal changes in other Blue systems. As with the analytical FPS approach, the outcome does not follow tactical expectations. Thus the Bonder approach, while producing the most reasonable system value changes, also produces a force ratio counterintuitive to military planners. Therefore output requires more analytical effort before a final product is available to the client/user.

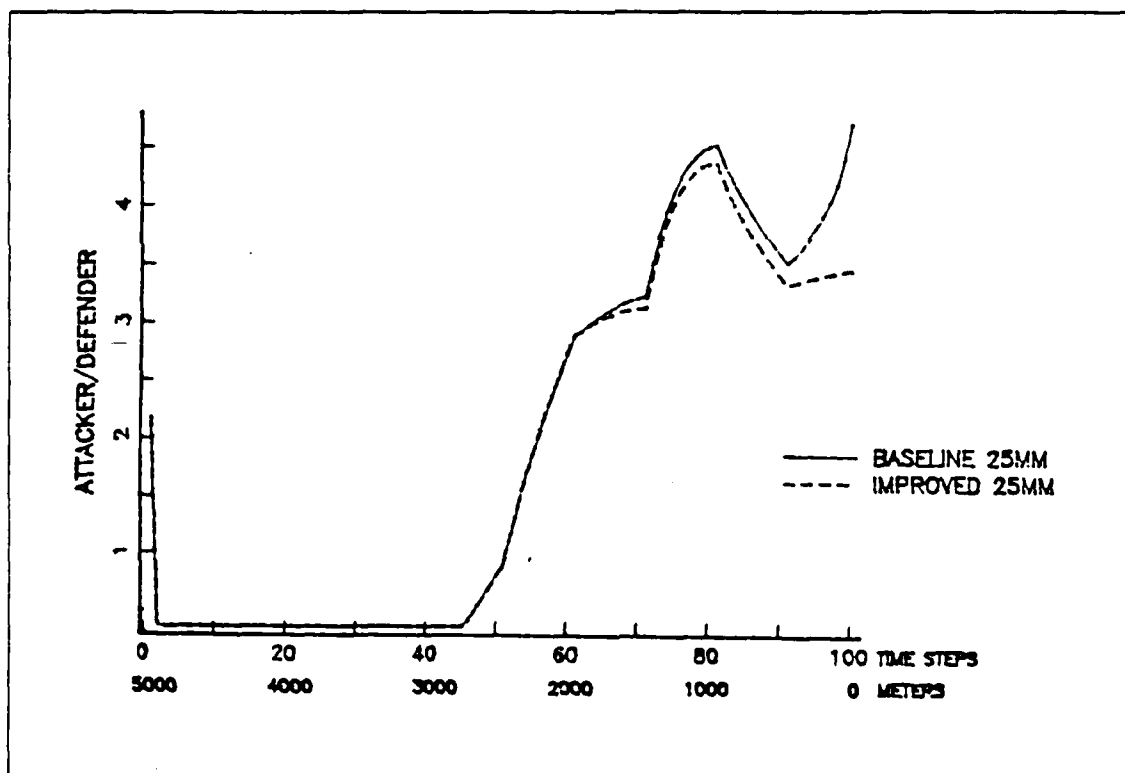


Figure 5.3 Force Ratio Changes in Bonder P-AP Model.

E. FORCE SIZE - LARGE FORCE BATTLE SIMULATION

A brief examination of the effect of increasing the number of forces on each side was conducted to determine if any problems arise within the various approaches. The original Blue force battalion-sized task force was expanded to a 2 battalion task force with 124 individual vehicles/systems. The Red force was enlarged to a understrengthened motorized rifle regiment with an assigned strength of 104 vehicles.

Results from the increased force size were consistent with the expectations derived from the previous simulation outputs. The subjective FPS displayed the same linear relationships with the battle termination going to the maximum allowable time/distance. The end result was a Blue victory with several Red force tanks, ATGMs and BMPs surviving the battle. The analytical FPS results were reversed with the Red force winning the battle in approximately 15 minutes of combat. This corresponds with the significant increase of ATGM systems found with the Red force and the tendency of the analytical approach to produce high attrition estimates for the respective systems. The final simulation was run using the Bonder attrition coefficient generation process and resulted in a full 20 minute attack with several Red force ATGMs surviving at battle termination. There was no indication that any methodology was significantly affected by the change in force size. Therefore, it is felt that division-size and larger force organizations are not expected to create any additional problems beyond those noted in the previous chapters.

F. SUMMARY

The individual outputs representing the three attrition generation methodologies indicated variations similar to those observed in Chapter IV. The subjective approach produces strictly linear changes when used in the Lanchester Square Law model and retains the proportional relationships established by the firepower scores. Within the P-AP model the linear outputs remain but changes between the systems are inconsistent with the physical modifications. This manifests itself in increased opposing force values and force ratio changes in the opposite direction to force/weapon system improvement. Further, changes are not consistent across opposing force artillery systems and therefore cause the approach to be considered unreliable for use within the P-AP model environment.

The results of the analytical FPS simulation indicate this approach is the most sensitive to change and results in the largest observed changes to system values. Within the Lanchester model, the approach provided attrition results consistent with

system upgrades. However, values computed in the P-AP model tend to overestimate increases in the relative value of opposing force systems. Consequently, force ratio predictions do not provide the user an interpretation that is intuitively consistent with force upgrades.

The Bonder approach continued to produce the most consistent and believable system values indicating that the technique provides a more balanced and credible response to system modification than either FPS approach. The low level of change in unmodified Blue force systems correlates with expected system value changes when physical parameters of systems are unchanged. Red force systems relative values display more realistic increases than in the analytical FPS method, although they are not considered to be intuitive outside the confines of the P-AP model. While certain aspects of the Bonder attrition coefficient methodology produce results that are considered counterintuitive to the military planner, examination and analysis of model output, indicate that the approach is well suited for use in aggregated models and the associated areas of analysis.

VI. SUMMARY AND RECOMMENDATIONS

Throughout the previous chapters the various facets of the subjective, analytical and Bonder attrition generation techniques have been examined at each phase of analysis. By allowing the simultaneous analysis of the various factors, it has been possible to see where the different methodologies succeed or fail to portray the dynamics of combat within the specified scenario. This side-by-side comparison likewise, extends to the interpretation of model output. By identifying these various areas, it is hoped that similar shortcomings in aggregated combat models, based on these attrition methodologies, can be avoided in the future. To this end, a summary of the salient points relevant to each methodology is presented below.

A. SUBJECTIVE FIREPOWER SCORES

The information developed through the examination, evaluation, and analysis of available data indicates that the use of the subjective FPS approach as a basis for the development of attrition coefficients is inappropriate. While a number of factors leading to this conclusion have been cited, they relate to several key areas. The first and foremost detractor from using the ratios of firepower scores/indices is the failure of the process to produce a casualty rate as defined by Lanchester. The use of a dimensionless scalar quantity as a surrogate attrition coefficient does not meet the basic requirement for dimensional consistency. The next factor of consequence is the use of a static value to represent a time-dependent process. This creates linear conditions where nonlinear attrition processes exist. Consequently, the linear state created by this approach produces conditions that are inconsistent with empirical data. This manifests itself in such areas as under/overestimation of attrition rates, output that fails to reflect or account for system capabilities (e.g. maximum effective range of weapons and their effect on the battle), and violation of the law of diminishing returns. Finally, the combination of these factors produces conditions that may lead to erroneous conclusions about system contributions to unit mission accomplishment and the effect of weapon improvement/replacement programs.

The supporters of subjective firepower scores will argue that the methodology does account for the quantifiable facets of combat and add that the method goes further by including technically undefined factors into the final system/unit value.

Further, the techniques provide a valid means to measure the relative values of weapons and units by the incorporation of those factors that influence combat that the mathematically based techniques ignore. This point of view is both valid and acceptable in the context of the use of subjective firepower scores as an assessor of relative worth. However, as a surrogate for an attrition coefficient, analysis indicates that subjective firepower scores are totally unacceptable and should not be used when developing attrition coefficients. This is not to say that subjective firepower scores do not have a place in the combat scoring system. Rather, as pointed out by Parry [Ref. 1: p.29], that while firepower scores are not appropriate for use as attrition generators, they may be used within specific contexts as interpreters of output data.

B. ANALYTICAL FIREPOWER SCORES

The use of the analytical firepower score approach for attrition coefficient generation corrects the majority of the deficiencies observed in the subjective FPS approach. The methodology provides a dimensionally correct value for a casualty rate and creates a predominantly acceptable portrayal of combat dynamics in the Lanchester and P-AP models. While the specific methodology uses only two variables, it reflects variations in the attrition rates and system values at the points where system changes occur, therefore allowing the user a unique snapshot of the battlefield at any given time step throughout the simulation. Overall, the approach provides the user a computational methodology that is uncomplicated and which produces easily interpretable output. However, this simplicity has its drawbacks.

While it was observed that the analytical FPS approach generally produces consistent results in both the Lanchester and P-AP model, the instances when these values deviate from the acceptable range (between 0.0 and 20.0 in this particular scenario) produces questions about the overall validity of the methodology. Within the Lanchester model, the tendency of the analytical approach was to produce results which were higher or lower than the other approaches, strongly suggesting that the technique is an under/overestimator. This obviously could lead to misinterpretation of simulation outputs if taken in isolated cases. Analysis of the associated values in the P-AP model indicates that the analytical FPS coefficients may produce intuitively questionable or incorrect values at various points in the simulation. This, in turn, leads the analyst and user to question the validity of the remaining data. As such, the use of this methodology may create undesirable discontinuities in output or require alternative and time consuming analytical techniques to extract meaningful information for the product user.

Since one of the goals was to determine the ability of the respective attrition techniques to acceptably portray the perceived dynamics of combat, the results from analysis of the analytical FPS approach in the P-AP model indicates sufficient discrepancies to warrant precluding its use. However, results from the Lanchester Square Law model suggest the methodology presents a viable alternative to far more complex generation methods without sacrificing accuracy in the model output. While the analytical techniques has its shortfalls, it appears that it can be used effectively in the Lanchester based models.

C. BONDER EQUATION ATTRITION COEFFICIENTS

Of the methodologies discussed and analyzed, the Bonder approach is the most sophisticated and input intensive. Consequently, initial expectations are that the approach should produce a more accurate and acceptable portrayal of combat dynamics. Analysis of the associated model outputs indicates that the Bonder approach meets these expectations. Outputs in both Lanchester and Potential-Antipotential models are the most consistent and believable of the three approaches.

The resulting information/data produced through use of the Bonder equation resulted in model outputs that meet intuitive expectations and and empirical data. The discontinuity of acceptable system values noted in the analytical FPS technique was not observed in the Bonder output. More importantly, the resulting values correlated with the intuitive expectations for each model and maintained a cross-model consistency unmatched by either FPS technique. Further, investigation of output from the P-AP model that appear to be counterintuitive to the military client reveals that the results are within all applicable model assumptions and conditions and are analytical correct. Consequently there is sufficient information to indicate that the Bonder equation technique for attrition coefficient generation provides acceptable portrayal of the combat process in either Lanchester and Potential-Antipotential simulation models.

D. RECOMMENDATIONS FOR FURTHER STUDY

To this point, it has been determined that the general category of methodologies that are used to derive subjective firepower scores are inappropriate for the generation of surrogate attrition coefficients. Likewise, there exists evidence that the analytical FPS approach may not provide valid results in the P-AP model environment. In each case these results were obtained for a specific scenario and model assumptions. To this end, the analytical examination of attrition methodologies has only begun. As such,

the following is a partial list of possible research topic areas proposed as an extension to this study:

- Examination of methodologies under different scenario and model assumptions to include use of full scale operational models such as Joint Theatre Level Simulation (JTLS).
- Determination of conditions under which the analytical FPS methodology produces only over- or underestimates of attrition rates and system values.
- Examination of specific sensitivity performance parameters of the models using the Bonder equation generated attrition coefficients.
- Effect of using alternative ranking methodologies in lieu of the Potential-Antipotential (eigenvalue) approach.
- Determination of limiting constraints on the use of the analytical FPS or Bonder equation techniques in aggregate combat models (i.e. under what conditions do these approaches no longer provide empirically or intuitively unacceptable results).

APPENDIX A

ANALYTICAL AND SUBJECTIVE FPS SIMULATION MODEL

The program listed below was developed to model the aggregation process for combat at the battalion-level. It is meant to be scenario specific and are not to be construed as an attempt to reflect the output of any other aggregate model. The program allows the user to follow the course of battle under the Lanchester Square Law processes as well as computes the relative value of the weapon systems at each time phase by using the Potential-Antipotential Method. The output should not be considered for real world purpose due to its oversimplification of the combat process. Its design objective was to provide a means to examine attrition coefficients in aggregate combat models.

```

      V COMBAT:A,B;Z,I;J,K;AINC,BINC;X,Y,XL,YL
[1]  A THE FOLLOWING VARIABLES MUST BE GLOBALLY AVAILABLE IN THE WORK SPACE
[2]  A AT AN ARRAY OF ALL AIJ MATRICES
[3]  A BT AN ARRAY OF ALL BJI MATRICES
[4]  A DIMENSIONS OF AT AND BT[NUMBER OF AIJ MATRICES;NUM OF SYS;NUM OF SYS]
[5]  A N A VECTOR CONTAINING THE NUMBER OF TIME STEPS FOR EACH AIJ OR BJI
[6]  A XI A VECTOR CONTAINING THE STARTING FORCE SIZE OF X
[7]  A YI A VECTOR CONTAINING THE STARTING FORCE SIZE OF Y
[8]  A DIMENSION OF XI AND YI [ NUMBER OF SYSTEMS ]
[9]  A XBP THE PERCENT OF INITIAL FORCE SIZE AT WHICH X SURRENDERS
[10] A YBP THE PERCENT OF INITIAL FORCE SIZE AT WHICH Y SURRENDERS
[11] A XBP AND YBP ARE VECTORS CONTAINING A VALUE FOR EACH SYSTEM
[12] A ASSIGNING A NEGATIVE VALUE TO A SYSTEM BREAK POINT WILL ALLOW ITS
[13] A FORCE SIZE TO BE DRIVEN TO 0 WITHOUT TERMINATING THE SIMULATION
[14] A THE NUMBER OF X SYSTEMS AND THE NUMBER OF Y SYSTEMS MUST BE EQUAL
[15] A THE FOLLOWING FUNCTIONS MUST ALSO BE PRESENT IN THE WORKSPACE TO
[16] A COMBAT - POTEN AND EIGENR
[17]
[18] K+1
[19] I+1
[20] Z+((pAT)[2],+/(N)
[21] SXT+SYT+XF+YF+Zp0
[22] F+VX+VY+10
[23] X+XI
[24] Y+YI
[25] A+B+((pAT)[2],(pAT)[2])p1E-10
[26] →L9
[27] L1:A+AT[I;:]
[28] B+BT[I;:]
[29] L9:J+0
[30] →(I>1)/L2
[31] AINC+(AT[1;:]-A)+N[1]
[32] BINC+(BT[1;:]-B)+N[1]
[33] →L3
[34] L2:AINC+(AT[I;:]-AT[(I-1);:])+N[I]
[35] BINC+(BT[I;:]-BT[(I-1);:])+N[I]
[36] L3:A POTEN B
[37] SXT[;K]+SX
[38] SYT[;K]+SY
[39] VX+VX,SX+.×XI
[40] VY+VY,SY+.×YI
[41] F+F,((SX+.×XI)+(SY+.×YI))

```

```

[42] XL←A+.×Y
[43] YL←B+.×X
[44] X←X-XL
[45] Y←Y-YL
[46] X←(X≥0)×X
[47] Y←(Y≥0)×Y
[48] XF[:K]←X
[49] YF[:K]←Y
[50] NTS←K
[51] K←K+1
[52] A←A+AINC
[53] B←B+BINC
[54] J←J+1
[55] →((+/(X+XI)≤XBP))≥1)/L5
[56] →L7
[57] L5:'Y WINS THE LANCHESTER BATTLE IN ',(NTS),' TIME STEPS'
[58] →L4
[59] L7:→((+/(Y+YI)≤YBP))≥1)/L6
[60] →L8
[61] L6:'X WINS THE LANCHESTER BATTLE IN ',(NTS),' TIME STEPS'
[62] →L4
[63] L8:→(J<N[I])/L3
[64] I←I+1
[65] →(I≤ρN)/L1
[66] L4:'
[67] '
[68] '      X FORCE SIZE'
[69] '
[70] 'SYSTEM  INITIAL      FINAL'
[71] C1←53ρ'M1ITVM21558IN'
[72] C2←53ρ'T72AT5122BMP152'
[73] C1,⊖(164⊖(51ρXI)),⊖(164⊖(51ρXF[:NTS]))
[74] '
[75] '
[76] '      Y FORCE SIZE'
[77] '
[78] 'SYSTEM  INITIAL      FINAL'
[79] '
[80] C2,⊖(164⊖(51ρYI)),⊖(164⊖(51ρYF[:NTS]))
[81] XF←XI,[2]XF
[82] YF←YI,[2]YF
[83] '
[84] 'SIMULATION COMPLETE'

```

The program POTEN uses a series of subroutines available in the NPS IMSL Library to compute the eigenvalues from the respective attrition matrices of the Blue and Red forces. These values are then incorporated into the Potential-antipotential Method to compute a series of relative weapon values to be output for later analysis.

```

      V A POTEN B;ABT;BAT;I;X;ABTS
[1] ABTS←(10*4)×ABT+Q(A+.×B)
[2] BAT←BAT+Q(B+.×A)
[3] EIGV←0.0001×EIGV+[(EIGENRABTS)[1;]]
[4] I←EIGV×((ρABT)ρ(1,((ρABT)[1])ρ0))
[5] ABT←ABT-I
[6] SX←1,⊖((- (ρABT)-1)×ABT))+.×(1+(-ABT[:1]))
[7] BAT←BAT-I
[8] BAT[1;]←B[:1]
[9] X←(EIGV×0.5),((ρBAT)[2]-1)ρ0)
[10] SY←(BAT)+.×X

```

APPENDIX B

BONDER SIMULATION MODEL

The following program is a modification of the aggregate model presented in Appendix A. It utilizes a series of matrices and vectors that represent the variables identified in Eqn. 2.4 and calculates attrition coefficients at each model time step. These matrices are then input to the Lanchester Square Law simulation and Potential-Antipotenial model. The output is a simulated flow of battle and the relative values of each weapon system based on the M1 tank.

```

VCOMBAT[[]]
V COMBAT; Z; I; J; K; AINC; BINC; X; Y; XL; YL
[1] A THE FOLLOWING VARIABLES MUST BE GLOBALLY AVAILABLE IN THE WORK SPACE
[2] A APKH, APHH, APHM, AP1 ARRAYS USED TO GENERATE ALL AIJ MATRICES
[3] A BPKH, BPHH, BPHM, BP1 ARRAYS USED TO GENERATE ALL BJI MATRICES
[4] A DIMENSIONS OF APKH AND BPKH [NUMBER OF AIJ MATRICES; NUM OF SYS; NUM OF
    SYS]
[5] A N A VECTOR CONTAINING THE NUMBER OF TIME STEPS FOR EACH MATRIX
[6] A XI A VECTOR CONTAINING THE STARTING FORCE SIZE OF X
[7] A YI A VECTOR CONTAINING THE STARTING FORCE SIZE OF Y
[8] A DIMENSION OF XI AND YI [NUMBER OF SYSTEMS]
[9] A XBP THE PERCENT OF INITIAL FORCE SIZE AT WHICH X SURRENDERS
[10] A YBP THE PERCENT OF INITIAL FORCE SIZE AT WHICH Y SURRENDERS
[11] A XBP AND YBP ARE VECTORS CONTAINING A VALUE FOR EACH SYSTEM
[12] A ASSIGNING A NEGATIVE VALUE TO A SYSTEM BREAK POINT WILL ALLOW ITS
[13] A FORCE SIZE TO BE DRIVEN TO 0 WITHOUT TERMINATING THE SIMULATION
[14] A THE NUMBER OF X SYSTEMS AND THE NUMBER OF Y SYSTEMS MUST BE EQUAL
[15] A THE FOLLOWING FUNCTIONS MUST ALSO BE PRESENT IN THE WORKSPACE TO
[16] A COMBAT - POTEN AND EIGENR
[17] A ROW MATRICES FOR FIRING TIMES T1, TH, TM (FOR BOTH SIDES) AND ROW
[18] A MATRICES FOR WEAPON SYSTEM MUZZLE VELOCITIES.
[19] A AN INITIAL RANGE VALUE, R MUST ALSO BE ENTERED WHICH CORRESPONDS
[20] A TO THE INPUT MATRICES.
[21] ' '
[22] A INITIALIZE VALUES, COUNTERS AND DIMENSION MATRICES
[23] ' '
[24] K+1
[25] I+1
[26] Z+((pAPKH)[2],+ /N)
[27] SXT+SYT+XF+YF+Zp0
[28] F+VX+VY+10
[29] X+XI
[30] Y+YI
[31] R+5000
[32] A+B+A1+B1+A2+B2+((pAPKH)[2],(pAPKH)[2])p0
[33] APK+APH+APM+AP1+BPK+BPH+BPM+BP1+((pAPKH)[2],(pAPKH)[2])p1E-10
[34] +L9
[35] ' '
[36] A SET VALUES FOR PK, PH, PM, P1 MATRICES NEEDED TO CALCULATE A AND B
[37] A AND INCREMENTATION PARAMETERS
[38] ' '
[39] L1:APK+APKH[(I-1);:]
[40] APH+APHH[(I-1);:]
[41] APM+APHM[(I-1);:]
[42] AP1+AP1H[(I-1);:]
[43] BPK+BPKH[(I-1);:]

```



```

44] BPH←BPHH[(I-1);:]
45] BPM←BPHM[(I-1);:]
46] BP1←BP1H[(I-1);:]
47] L9:J←0
48] →(I>1)/L2
49] APKINC←(APKH[1;]-APK)+N[1]
50] APHINC←APMINC←(APHH[1;]-APH)+N[1]
51] AP1INC←(AP1H[1;]-AP1)+N[1]
52] BPKINC←(BPKH[1;]-BPK)+N[1]
53] BPHINC←BPMINC←(BPHH[1;]-BPH)+N[1]
54] BP1INC←(BP1H[1;]-BP1)+N[1]
55] →L3
56] L2:APKINC←(APKH[I;]-APKH[(I-1);])+(N[I])
57] APHINC←APMINC←(APHH[I;]-APHH[(I-1);])+(N[I])
58] AP1INC←(AP1H[I;]-AP1H[(I-1);])+(N[I])
59] BPKINC←(BPKH[I;]-BPKH[(I-1);])+(N[I])
60] BPHINC←BPMINC←(BPHH[I;]-BPHH[(I-1);])+(N[I])
61] BP1INC←(BP1H[I;]-BP1H[(I-1);])+(N[I])
62]
63] a CALCULATE A AND B ATTRITION MATRICES
64]
65] L3:A[;1]←((1-APH)[;1])÷Q(APK[;1])
66] A[;2]←((1-APH)[;2])÷Q(APK[;2])
67] A[;3]←((1-APH)[;3])÷Q(APK[;3])
68] A←(A+APH)-AP1
69] ATF←1.5p(R+AV)
70] BTF←1.5p(R+BV)
71] A1[;1]←((ATH+ATF)[;1])÷Q(APK[;1])
72] A1[;2]←((ATH+ATF)[;2])÷Q(APK[;2])
73] A1[;3]←((ATH+ATF)[;3])÷Q(APK[;3])
74] A2[;1]←((ATM+ATF)[;1])÷Q(APM[;1])
75] A2[;2]←((ATM+ATF)[;2])÷Q(APM[;2])
76] A2[;3]←((ATM+ATF)[;3])÷Q(APM[;3])
77] A[;1]←(A2[;1]×A[;1])+A1[;1]+((AT1-ATH)[;1])
78] A[;2]←(A2[;2]×A[;2])+A1[;2]+((AT1-ATH)[;2])
79] A[;3]←(A2[;3]×A[;3])+A1[;3]+((AT1-ATH)[;3])
80] A[;4]←(0.5×APK[;4])
81] A[;5]←(0.5×APK[;5])
82] A←1+A
83] B[;1]←((1-BPH)[;1])÷Q(BPK[;1])
84] B[;2]←((1-BPH)[;2])÷Q(BPK[;2])
85] B[;3]←((1-BPH)[;3])÷Q(BPK[;3])
86] B←(B+BPH)-BP1
87] B1[;1]←((BTH+BTF)[;1])÷Q(BPK[;1])
88] B1[;2]←((BTH+BTF)[;2])÷Q(BPK[;2])
89] B1[;3]←((BTH+BTF)[;3])÷Q(BPK[;3])
90] B2[;1]←((BTM+BTF)[;1])÷Q(BPM[;1])
91] B2[;2]←((BTM+BTF)[;2])÷Q(BPM[;2])
92] B2[;3]←((BTM+BTF)[;3])÷Q(BPM[;3])
93] B[;1]←(B2[;1]×B[;1])+B1[;1]+((BT1-BTH)[;1])
94] B[;2]←(B2[;2]×B[;2])+B1[;2]+((BT1-BTH)[;2])
95] B[;3]←(B2[;3]×B[;3])+B1[;3]+((BT1-BTH)[;3])
96] B[;4]←(0.5×BPK[;4])
97] B[;5]←(0.5×BPK[;5])
98] B←1+B
99] B[;5]←(0.25×BPK[;5])
100] B←1+B
101] A POTEN B
102] SXT[;K]←SX
103] SYT[;K]←SY
104] VX←VX,SX+.×XI
105] VY←VY,SY+.×YI
106] F←F,((SX+.×XI)+(SY+.×YI))
107] XL←A+.×Y
108] YL←B+.×X
109] X←X-XL
110] Y←Y-YL
111] X←(X≥0)×X
112] Y←(Y≥0)×Y
113]

```

```

[114] XF[:K]←X
[115] YF[:K]←Y
[116] NTS←K
[117] K←K+1
[118] APK←APK+APKINC
[119] APH←APM+APH+APHINC
[120] AP1←AP1+AP1INC
[121] BPK←BPK+BPKINC
[122] BPH←BPH+BPHINC
[123] BPM←BPM+BPMINC
[124] BP1←BP1+BP1INC
[125] R←R-50
[126] →(R≤500)/L15
[127] →L20
[128] →(R≤0)/L6
[129] L15:BV←1 5 p 1500 200 1345 327 295
[130] BTM←BTH←1 5 p 6 60 2 30 60
[131] BT1←1 5 p 8 60 8 30 60
[132] L20:J←J+1
[133] →((+/(X+XI)≤XBP))≥1)/L5
[134] →L7
[135] L5:'Y WINS THE LANCHESTER BATTLE IN ',(NTS),' TIME STEPS AT ',(R),' ME
    ERS'
[136] →L4
[137] L7:→((+/(Y+YI)≤YBP))≥1)/L6
[138] →L8
[139] L6:'X WINS THE LANCHESTER BATTLE IN ',(NTS),' TIME STEPS AT ',(R),' ME
    ERS'
[140] →L4
[141] L8:→(J<N[I])/L3
[142] I←I+1
[143] →(I≤pN)/L1
[144] L4:' '
[145] ' '
[146] ' X FORCE SIZE'
[147] ' '
[148] 'SYSTEM INITIAL FINAL'
[149] C1←5 3 p'M1 ITVM2 1558IN'
[150] C2←5 3 p'T72AT5BMP122152'
[151] C1,*(16 4 *(5 1 pXI)),*(16 4 *(5 1 pXF[:NTS]))
[152] ' '
[153] ' '
[154] ' Y FORCE SIZE'
[155] ' '
[156] 'SYSTEM INITIAL FINAL'
[157] ' '
[158] C2,*(16 4 *(5 1 pYI)),*(16 4 *(5 1 pYF[:NTS]))
[159] XF+XI,[2] XF
[160] YF+YI,[2] YF
[161] ' '
[162] 'SIMULATION COMPLETE'

```

The subroutine POTEN called upon in this program is used to compute the eigenvalues and generate the values necessary to determine the relative values of each weapon system compared to the M1 Tank.

```

      7POTEN[007
      7A POTEN B:ABT:BAT:I:X:ABTS
[1] ABTS←(10*4)*ABT+*(A+.*E)
[2] BAT←BAT+*(E+.*A)
[3] EIGV←0.0001*EIGV+*((EIGEN ABTS)[1;])
[4] I←EIGV*((cABT)p(1,((cABT)[1;])p0))
[5] ABT←ABT-I

```

NO-A106 964

COMPARATIVE ANALYSIS OF ATTRITION GENERATION
METHODOLOGIES UTILIZED IN AGGREGATE COMBAT MODELS(U)
NAVAL POSTGRADUATE SCHOOL MONTEREY CA K L MURPHY

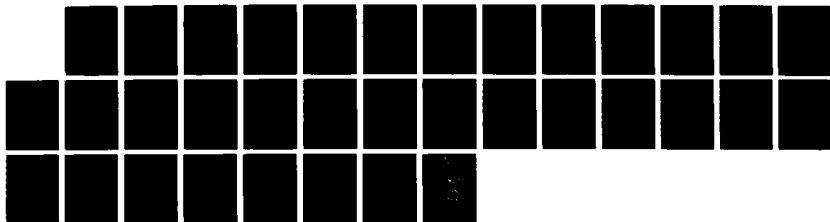
2/2

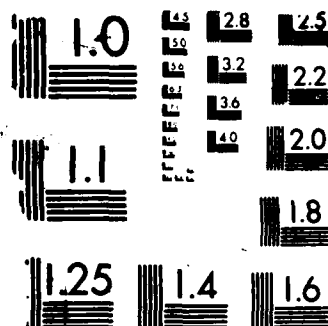
UNCLASSIFIED

SEP 87

F/G 12/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

```

[6] SX+1, (8((- (pABT)-1)+ABT))+.x(1+(-ABT[:1]))
[7] BAT+BAT-I
[8] BAT[:1]+B[:1]
[9] X+(EIGV*0.5), ((pBAT)[2]-1)p0
[10] SY+(8BAT)+.xX

```

APPENDIX C

ANALYTICAL FPS COMBAT RESULT

The following data was developed by using ammunition expenditure values and average probability of killing any target to calculate firepower scores. The method is a simplified analytical firepower scoring methodology and is present for its supporting information value and not as a desired technique for generation of weapon scores. Based on the firepower scores presented in Chapter 3, Section D, the relative worth matrices are:

TABLE 18
RELATIVE WORTH MATRICES

	M1	ITV	M2	155	203
T72	2.0000	4.2424	4.2424	15.5555	15.5555
AT5	0.5714	1.2121	1.2121	4.4444	4.4444
BMP	0.7142	1.5151	1.5151	5.5555	5.5555
155	0.1285	0.2727	0.2727	1.0000	1.0000
203	0.1285	0.2727	0.2727	1.0000	1.0000

	T72	AT5	BMP	122	152
M1	0.5000	1.7500	1.4000	7.7821	7.7821
ITV	0.2357	0.8250	0.8259	3.6670	3.6670
M2	0.2357	0.8250	0.8259	3.6670	3.6670
155	0.0642	0.2250	0.1800	1.0000	1.0000
203	0.0642	0.2250	0.1800	1.0000	1.0000

The associated attrition matrices based on the ammunition expenditures and average probability of kill equations, in their initial weights and adjusted to fit the simulation scenario (no direct fire weapon engagement of artillery) are shown in Table 19. The attrition matrices were then entered into the simulation model. The results from the base attrition coefficient matrices are shown in Table 20 .

In order to reduce the extremely high pace of battle, the attrition matrices were reduced by a factor of 100 ($A_{ij} \times 0.01$). The results of the subsequent simulation run are shown in Table 21 .

TABLE 19
INITIAL ATTRITION COEFFICIENTS

	A_{ij}				
	T72	AT5	BMP	122	152
M1	2.0000	0.5714	0.7142	0.1285	0.1285
ITV	4.2424	1.2121	1.5151	0.2727	0.2727
M2	4.2424	1.2121	1.5151	0.2727	0.2727
155	0.0000	0.0000	0.0000	0.0000	1.0000
203	0.0000	0.0000	0.0000	0.0000	1.0000

	B_{ij}				
	M1	ITV	M2	155	203
T72	0.5000	0.2357	0.2357	0.0642	0.064200
AT5	1.7500	0.8250	0.8250	0.2250	0.2250
BMP	1.4000	0.6600	0.6600	0.1800	0.1800
122	0.0000	0.0000	0.0000	1.0000	1.0000
152	0.0000	0.0000	0.0000	0.0000	1.0000

TABLE 20
SIMULATION RESULTS FOR INITIAL MATRICES

X WINS THE LANCHESTER BATTLE IN 3 TIME STEPS

X FORCE SIZE			Y FORCE SIZE		
SYSTEM	INITIAL	FINAL	SYSTEM	INITIAL	FINAL
M1	42.0000	38.4924	T72	4.0000	.0000
ITV	10.0000	2.5589	AT5	10.0000	.0000
M2	6.0000	.0000	BMP	10.0000	.0000
155	6.0000	4.2800	122	6.0000	3.2400
8IN	4.0000	2.2800	152	6.0000	4.9200

Following analysis of the second simulation run, the pace was still too fast and an additional reduction of the attrition coefficients was undertaken. The subsequent reduction of matrices by 2 produced matrices that had values equal to 0.005 of the original. Simulation results for the attrition coefficients are displayed in Table 22.

TABLE 21
SIMULATION RESULTS FOR 0.01 REDUCED MATRICES

X WINS THE LANCHESTER BATTLE IN 3 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	41.9388	T72	4.0000	3.9238
ITV	10.0000	9.8701	AT5	10.0000	.0000
M2	6.0000	5.8701	BMP	10.0000	9.7867
155	6.0000	5.9820	122	6.0000	5.9700
203	4.0000	3.9820	152	6.0000	5.9880

X WINS THE LANCHESTER BATTLE IN 21 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.5284	T72	4.0000	.2773
ITV	10.0000	6.8781	AT5	10.0000	.0000
M2	6.0000	2.8781	BMP	10.0000	.0000
155	6.0000	5.1122	122	6.0000	4.5819
203	4.0000	3.1122	152	6.0000	5.4459

X WINS THE LANCHESTER BATTLE IN 23 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.4963	T72	4.0000	.0000
ITV	10.0000	6.8100	AT5	10.0000	.0000
M2	6.0000	2.8100	BMP	10.0000	.0000
155	6.0000	5.0036	122	6.0000	4.4185
203	4.0000	3.0036	152	6.0000	5.3842

The resulting pace of battle remained higher than dictated by the scenario conditions should have allowed. Consequently another reduction of the attrition coefficient values to .0025 of the base values was executed. This final reduction produced a battle termination after 18.25 minutes and a closing distance of 1350 meters. The simulated flow for the final attrition matrices are in Table 23 .

TABLE 22
SIMULATION RESULTS FOR .005 REDUCTION

X WINS THE LANCHESTER BATTLE IN 4 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	41.9410	T72	4.0000	3.9239
ITV	10.0000	9.8748	AT5	10.0000	.0000
M2	6.0000	5.8748	BMP	10.0000	9.7868
155	6.0000	5.9820	122	6.0000	5.9700
203	4.0000	3.9820	152	6.0000	5.9880

X WINS THE LANCHESTER BATTLE IN 36 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.5661	T72	4.0000	.3361
ITV	10.0000	6.9580	AT5	10.0000	.0000
M2	6.0000	2.9580	BMP	10.0000	.0000
155	6.0000	5.1269	122	6.0000	4.6061
8IN	4.0000	3.1269	152	6.0000	5.4556

X WINS THE LANCHESTER BATTLE IN 39 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.5402	T72	4.0000	.0000
ITV	10.0000	6.9031	AT5	10.0000	.0000
M2	6.0000	2.9031	BMP	10.0000	.0000
155	6.0000	5.0453	122	6.0000	4.4831
203	4.0000	3.0453	152	6.0000	5.4091

TABLE 23
SIMULATION RESULTS FOR .0025 REDUCTION

X WINS THE LANCHESTER BATTLE IN 5 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	41.9505	T72	4.0000	3.9365
ITV	10.0000	9.8950	AT5	10.0000	.0000
M2	6.0000	5.8950	BMP	10.0000	9.8223
155	6.0000	5.9850	122	6.0000	5.9750
203	4.0000	3.9850	152	6.0000	5.9900

X WINS THE LANCHESTER BATTLE IN 65 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.5917	T72	4.0000	.4235
ITV	10.0000	7.0123	AT5	10.0000	.0000
M2	6.0000	3.0123	BMP	10.0000	.0000
155	6.0000	5.1480	122	6.0000	4.6391
203	4.0000	3.1480	152	6.0000	5.4683

X WINS THE LANCHESTER BATTLE IN 73 TIME STEPS

SYSTEM	X FORCE SIZE		SYSTEM	Y FORCE SIZE	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	40.5571	T72	4.0000	.0000
ITV	10.0000	6.9390	AT5	10.0000	.0000
M2	6.0000	2.9390	BMP	10.0000	.0000
155	6.0000	5.0391	122	6.0000	4.4751
8IN	4.0000	3.0391	152	6.0000	5.4063

APPENDIX D

ANALYTICAL FPS ATTRITION MATRICES

The following is a listing of the attrition coefficient matrices, generated using Eqn. 3 and used in the aggregated simulation of a Blue Tank Heavy Task Force attacking a Reinforced Red Company Team in a strongpoint defense. Matrices listed under A_{ij} represent that rate at which the Threat forces (Red) kill the Blue attacking systems. Matrices listed under B_{ji} represent that rate at which the attacking systems kill the Red defenders. Each matrix represents the values at the lower range indicated. By using linear interpolation the values of the attrition coefficients for ranges between any two matrices can be obtained.

Factors used to develop these values were probability of single shot kill, target acquisition rate, and system rates of fire. Acquisition rates for tanks allowed 0.6 of all available opposing tanks to be acquired and 0.4 of all other systems. IFV and ATGM systems limited by sectors of fire were given 0.4 acquisition rates. Artillery pieces were considered capable of engaging any system and therefore were given 1.0 acquisition rates. Previously identified artillery versus artillery engagement rules are reflected in the matrices. Rates of fire were based on the sustained rates of fire of each weapon under the three combat zones described in Chapter III. The resulting flow of battle to the point of weapon system extinction is shown in Table 26 .

TABLE 24

 A_{IJ}

5000-4500 Meters

	T72	AT5	122	BMP	152
M1	.000000	.000000	.000500	.000000	.000192
ITV	.000000	.000000	.003500	.000000	.001344
M2	.000000	.000000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

4500-4000 Meters

	T72	AT5	122	BMP	152
M1	.000000	.000000	.000500	.000000	.001920
ITV	.000000	.000000	.003500	.000000	.001344
M2	.000000	.000000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

4000-3500 Meters

	T72	AT5	122	BMP	152
M1	.000000	.000000	.000500	.000000	.001920
ITV	.000000	.000000	.035000	.000000	.001344
M2	.000000	.000000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

3500-3000 Meters

	T72	AT5	122	BMP	152
M1	.000000	.028800	.000500	.000000	.001920
ITV	.000000	.023200	.003500	.000000	.001344
M2	.000000	.024000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

3000-2500 Meters

	T72	AT5	122	BMP	152
M1	.014400	.064800	.000500	.000000	.001920
ITV	.030000	.054400	.003500	.000000	.001344
M2	.021600	.052000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

2500-2000 Meters

	T72	AT5	122	BMP	152
M1	.043200	.088800	.000500	.000000	.001920
ITV	.040000	.070400	.003500	.000000	.001344
M2	.048000	.068000	.002500	.000000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

2000-1500 Meters

	T72	AT5	122	BMP	152
M1	.055200	.088800	.000500	.000000	.001920
ITV	.059200	.070400	.003500	.018000	.001344
M2	.055200	.068000	.002500	.009000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

1500-1000 Meters

M1	.273600	.088800	.000500	.003000	.001920
ITV	.211200	.070400	.003500	.091200	.001344
M2	.206400	.068000	.002500	.084000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

1000-500 Meters

M1	.302400	.099600	.000500	.015000	.001920
ITV	.220800	.074400	.003500	.120000	.001344
M2	.216000	.072000	.002500	.150000	.000960
122	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

500 - 0 Meters

M1	.765000	.099600	.000500	.060000	.001920
ITV	.552000	.074400	.003500	.510000	.001344
M2	.540000	.072000	.002500	.600000	.000960
155	.000000	.000000	.000000	.000000	.002780
203	.000000	.000000	.000000	.000000	.002780

TABLE 25

 B_{IJ}

5000-4500 Meters

	M1	ITV	M2	155	203
T72	.000000	.000000	.000000	.000500	.000248
AT5	.000000	.000000	.000000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.000000	.000000	.000000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

4500-4000 Meters

T72	.000000	.000000	.000000	.000500	.000248
AT5	.000000	.000000	.000000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.000000	.000000	.000000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

4000-3500 Meters

T72	.000000	.000000	.000000	.000500	.000248
AT5	.000000	.000000	.000000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.000000	.000000	.000000	.002500	.001320
155	.000000	.000000	.000000	.000000	.001640

3500-3000 Meters

T72	.000000	.006000	.006000	.000500	.000248
AT5	.000000	.006000	.006000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.000000	.003000	.003000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

3000-2500 Meters

T72	.001000	.010000	.010000	.000500	.000248
AT5	.002000	.010000	.010000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.001000	.005000	.005000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

2500-2000 Meters

T72	.007500	.018000	.018000	.000500	.000248
AT5	.015000	.018000	.018000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.007500	.009000	.009000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

2000-1500 Meters

	M1	ITV	M2	155	203
T72	.010000	.020000	.020000	.000500	.000248
AT5	.020000	.020000	.020000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.010000	.010000	.010000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

1500-1000 Meters

T72	.030000	.020000	.020000	.000500	.000248
AT5	.060000	.020000	.020000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.030000	.010000	.010000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

1000- 500 Meters

T72	.030000	.020000	.020000	.000500	.000248
AT5	.060000	.020000	.020000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.030000	.010000	.010000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

500 - 0 Meters

T72	.105000	.020000	.001000	.000500	.000248
AT5	.210000	.020000	.150000	.003500	.001860
122	.000000	.000000	.000000	.003500	.001640
BMP	.105000	.010000	.075000	.002500	.001320
152	.000000	.000000	.000000	.000000	.001640

TABLE 26
ANALYTICAL FPS SIMULATION RESULTS
Y WINS THE LANCHESTER BATTLE IN 45 TIME STEPS

SYSTEM	X Force Size		SYSTEM	Y Force Size	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	35.1843	T72	4.0000	1.1954
ITV	10.0000	3.5753	AT5	10.0000	5.6126
M2	6.0000	0.0000	BMP	10.0000	7.5812
155	6.0000	5.3543	122	6.0000	4.9758
203	4.0000	3.3543	152	6.0000	5.7614

X WINS THE LANCHESTER BATTLE IN 53 TIME STEPS

SYSTEM	X Force Size		SYSTEM	Y Force Size	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	31.3864	T72	4.0000	0.0000
ITV	10.0000	0.1634	AT5	10.0000	3.0507
M2	6.0000	0.0000	BMP	10.0000	6.2582
155	6.0000	5.2266	122	6.0000	4.7842
203	4.0000	3.2266	152	6.0000	5.7182

Y WINS THE LANCHESTER BATTLE IN 54 TIME STEPS

SYSTEM	X Force Size		SYSTEM	Y Force Size	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	31.0802	T72	4.0000	0.0000
ITV	10.0000	0.0000	AT5	10.0000	2.4299
M2	6.0000	0.0000	BMP	10.0000	5.9426
155	6.0000	5.2107	122	6.0000	4.7606
203	4.0000	3.2107	152	6.0000	5.7129

X WINS THE LANCHESTER BATTLE IN 58 TIME STEPS

SYSTEM	X Force Size		SYSTEM	Y Force Size	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	30.4572	T72	4.0000	0.0000
ITV	10.0000	0.0000	AT5	10.0000	0.0000
M2	6.0000	0.0000	BMP	10.0000	4.5117
155	6.0000	5.1473	122	6.0000	4.6670
203	4.0000	3.1473	152	6.0000	5.6920

X WINS THE LANCHESTER BATTLE IN 71 TIME STEPS

SYSTEM	X Force Size		SYSTEM	Y Force Size	
	INITIAL	FINAL		INITIAL	FINAL
M1	42.0000	30.2860	T72	4.0000	0.0000
ITV	10.0000	0.0000	AT5	10.0000	0.0000
M2	6.0000	0.0000	BMP	10.0000	0.0000
155	6.0000	4.9426	122	6.0000	4.3721
203	4.0000	2.9426	152	6.0000	5.6269

APPENDIX E

BONDER EQUATION INPUT VALUES

The following is a breakdown of the major input values used by the Bonder Equation Model (Appendix B), to compute attrition coefficients. A change in the velocity vector for the M2 IFV occurs in the actual program rather than by calling a new vector from the associated APL workspace. All other values are presented as they are input to the simulation.

TABLE 27
RED FORCE INPUT VECTORS

Firing Cycle Times					
	T72	AT5	BMP	122	152
t_i	8	60	20	30	30
t_h	7	60	15	30	30
t_m	7	60	15	30	30

Times to fire the initial round (t_i) and the successive rounds following a miss (t_m) or hit (t_h) are in seconds.

Muzzle velocity of weapons (mps)				
T72	AT5	BMP	122	152
1700	200	700	523	497

TABLE 28
CONDITIONAL PROBABILITY TABLES FOR FOLLOW-ON HITS

Probability of hit given a hit or miss on the previous shot ($P(h|h)$, $P(h|m)$). Each matrix represents a 500 meter step from 4500-0 meters

	T72	AT5	BMP	122	152
M1	.00000	.00000	.00000	.50000	.50000
ITV	.00000	.00000	.00000	.50000	.50000
M2	.00000	.00000	.00000	.50000	.50000
155	.00000	.00000	.00000	.50000	.50000
203	.00000	.00000	.00000	.50000	.50000
	.00000	.00000	.00000	.50000	.50000
	.00000	.00000	.00000	.50000	.50000
	.00000	.00000	.00000	.50000	.50000
	.00000	.00000	.00000	.50000	.50000
	.00000	.00000	.00000	.50000	.50000
	.00000	.91000	.00000	.50000	.50000
	.00000	.93000	.00000	.50000	.50000
	.00000	.81000	.00000	.50000	.50000
	.00000	.91000	.00000	.50000	.50000
	.00000	.91000	.00000	.50000	.50000
	T72	AT5	BMP	122	152
	.00000	.91000	.00000	.50000	.50000
	.00000	.93000	.00000	.50000	.50000
	.00000	.81000	.00000	.50000	.50000
	.00000	.91000	.00000	.50000	.50000
	.00000	.91000	.00000	.50000	.50000
	.02500	.91000	.00000	.50000	.50000
	.01500	.93000	.00000	.50000	.50000
	.02000	.81000	.00000	.50000	.50000
	.02500	.91000	.00000	.50000	.50000
	.02500	.91000	.00000	.50000	.50000
	.05000	.91000	.10000	.50000	.50000
	.30000	.93000	.01000	.50000	.50000
	.40000	.81000	.05000	.50000	.50000
	.05000	.91000	.10000	.50000	.50000
	.05000	.91000	.10000	.50000	.50000
	T72	AT5	BMP	122	152
	.40000	.91000	.20000	.50000	.50000
	.38000	.93000	.20000	.50000	.50000
	.39000	.81000	.10000	.50000	.50000
	.40000	.91000	.20000	.50000	.50000
	.40000	.91000	.20000	.50000	.50000
	.90000	.91000	.55000	.50000	.50000
	.77000	.93000	.12500	.50000	.50000
	.83000	.81000	.27000	.50000	.50000
	.90000	.91000	.55000	.50000	.50000
	.90000	.91000	.55000	.50000	.50000

T72	AT5	BMP	122	152
.95000	.91000	.87000	.50000	.50000
.99000	.93000	.83500	.50000	.50000
.99000	.81000	.63000	.50000	.50000
.95000	.91000	.87000	.50000	.50000
.95000	.91000	.87000	.50000	.50000
.95000	.91000	.95000	.50000	.50000
.99000	.93000	.99000	.50000	.50000
.99000	.81000	.95000	.50000	.50000
.95000	.91000	.95000	.50000	.50000
.95000	.91000	.95000	.50000	.50000

TABLE 29
CONDITIONAL KILL PROBABILITIES FOR RED FORCE SYSTEMS

Probability of a kill given a hit occurs $[P(k|h)]$

	T72	AT5	BMP	122	152
M1	.00000	.00000	.00000	.00100	.00077
ITV	.00000	.00000	.00000	.00700	.00538
M2	.00000	.00000	.00000	.00500	.00384
155	.00000	.00000	.00000	.00700	.01112
203	.00000	.00000	.00000	.00700	.01112
	.00000	.00000	.00000	.00100	.00077
	.00000	.00000	.00000	.00700	.00538
	.00000	.00000	.00000	.00500	.00384
	.00000	.00000	.00000	.00700	.01112
	.00000	.00000	.00000	.00700	.01112
	.00000	.00000	.00000	.00100	.00077
	.00000	.00000	.00000	.00700	.00538
	.00000	.00000	.00000	.00500	.00384
	.00000	.00000	.00000	.00700	.01112
	.00000	.00000	.00000	.00700	.01112
	.00000	.48000	.00000	.00100	.00077
	.00000	.58000	.00000	.00700	.00538
	.00000	.60000	.00000	.00500	.00384
	.00000	.60000	.00000	.00700	.01112
	.00000	.60000	.00000	.00700	.01112
	.08000	.54000	.00000	.00100	.00077
	.25000	.68000	.00000	.00700	.00538
	.18000	.65000	.00000	.00500	.00384
	.18000	.65000	.00000	.00700	.01112
	.18000	.65000	.00000	.00700	.01112
	.24000	.74000	.00000	.00100	.00077
	.33300	.88000	.00000	.00700	.00538
	.40000	.85000	.00000	.00500	.00384
	.40000	.85000	.00000	.00700	.01112
	.40000	.85000	.00000	.00700	.01112
	.30670	.74000	.00000	.00100	.00077
	.49300	.88000	.15000	.00700	.00538
	.46000	.85000	.06000	.00500	.00384
	.46000	.85000	.06000	.00700	.01112
	.46000	.85000	.06000	.00700	.01112
	.76000	.74000	.01000	.00100	.00077
	.88000	.88000	.38000	.00700	.00538
	.86000	.85000	.28000	.00500	.00384
	.86000	.85000	.28000	.00700	.01112
	.86000	.85000	.28000	.00700	.01112
	.84000	.83000	.05000	.00100	.00077
	.92000	.93000	.50000	.00700	.00538
	.90000	.90000	.50000	.00500	.00384
	.90000	.90000	.50000	.00700	.01112
	.90000	.90000	.50000	.00700	.01112

T72	AT5	BMP	122	152
.84000	.83000	.08000	.00100	.00077
.92000	.93000	.85000	.00700	.00538
.90000	.90000	.80000	.00500	.00384
.90000	.90000	.80000	.00700	.01112
.90000	.90000	.80000	.00700	.01112

TABLE 30
BLUE FORCE INPUT VECTORS

	Firing Cycle Times				
	M1	ITV	M2	155	203
t_i	8	60	8	30	60
t_h	6	60	2	30	60
t_h	6	60	2	30	60

Times to fire the initial round (t_i) and the successive rounds following a miss (t_m) or hit (t_h) in seconds.

Muzzle Velocity of Weapon Systems

M1	ITV	M2	155	203
1500	200	1345	327	295

Probability of hit given a hit or miss on the previous shot ($P(h|h)$, $P(h|m)$). Each matrix represents a 500 meter step from 4500-0 meters

114

M1	ITV	M2	155	203
.60000	.60000	.89000	.50000	.50000
.60000	.20000	.20000	.50000	.50000
.70000	.28000	.28000	.50000	.50000
.60000	.60000	.89000	.50000	.50000
.60000	.60000	.89000	.50000	.50000

TABLE 32
CONDITIONAL KILL PROBABILITIES FOR BLUE FORCE SYSTEMS

Probability of a kill given a hit occurs $P(k|h)$

M1	ITV	M2	155	203
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.00000	.00000	.50000	.50000
.00000	.70000	.70000	.50000	.50000
.00000	.28000	.28000	.50000	.50000
.00000	.38000	.38000	.50000	.50000
.00000	.70000	.70000	.50000	.50000
.00000	.70000	.70000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.28000	.28000	.50000	.50000
.03000	.38000	.38000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.27000	.27000	.50000	.50000
.04500	.37000	.37000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.70000	.70000	.50000	.50000
.53000	.67500	.67500	.50000	.50000
.53000	.25500	.25500	.50000	.50000
.06000	.35500	.35500	.50000	.50000
.53000	.67500	.67500	.50000	.50000
.53000	.67500	.67500	.50000	.50000
.54500	.65000	.65000	.50000	.50000
.54500	.24000	.24000	.50000	.50000
.11000	.34000	.34000	.50000	.50000
.54500	.65000	.65000	.50000	.50000
.54500	.65000	.65000	.50000	.50000
.56000	.62000	.62000	.50000	.50000
.56000	.22000	.22000	.50000	.50000
.16000	.32000	.32000	.50000	.50000
.56000	.62000	.62000	.50000	.50000
.56000	.62000	.62000	.50000	.50000
.59000	.61000	.85000	.50000	.50000
.59000	.20000	.20000	.50000	.50000
.40000	.30000	.30000	.50000	.50000
.59000	.61000	.85000	.50000	.50000
.59000	.61000	.85000	.50000	.50000

M1	ITV	M2	155	203
.60000	.60000	.89000	.50000	.50000
.60000	.20000	.20000	.50000	.50000
.70000	.28000	.28000	.50000	.50000
.60000	.60000	.89000	.50000	.50000
.60000	.60000	.89000	.50000	.50000

APPENDIX F **BONDER ATTRITION COEFFICIENT BATTLE RESULTS**

TABLE 33
BONDER ATTRITION COEFFICIENT SIMULATION RESULTS

X WINS THE LANCHESTER BATTLE IN 52 TIME STEPS AT 2400 METERS

X FORCE SIZE			Y FORCE SIZE		
SYSTEM	INITIAL	FINAL	SYSTEM	INITIAL	FINAL
M1	42.0000	41.2256	T72	4.0000	0.0000
ITV	10.0000	6.4974	AT5	10.0000	7.6810
M2	6.0000	4.8138	BMP	10.0000	4.3676
155	6.0000	4.4826	122	6.0000	4.8973
203	4.0000	2.4826	152	6.0000	5.7520

X WINS THE LANCHESTER BATTLE IN 57 TIME STEPS AT 2150 METERS

X FORCE SIZE			Y FORCE SIZE		
SYSTEM	INITIAL	FINAL	SYSTEM	INITIAL	FINAL
M1	42.0000	40.8098	T72	4.0000	0.0000.
ITV -	10.0000	5.9243	AT5	10.0000	6.3674
M2	6.0000	4.6185	BMP	10.0000	0.0000
155	6.0000	4.3230	122	6.0000	4.8002
8IN	4.0000	2.3230	152	6.0000	5.7322

X WINS THE LANCHESTER BATTLE IN 79 TIME STEPS AT 1050 METERS

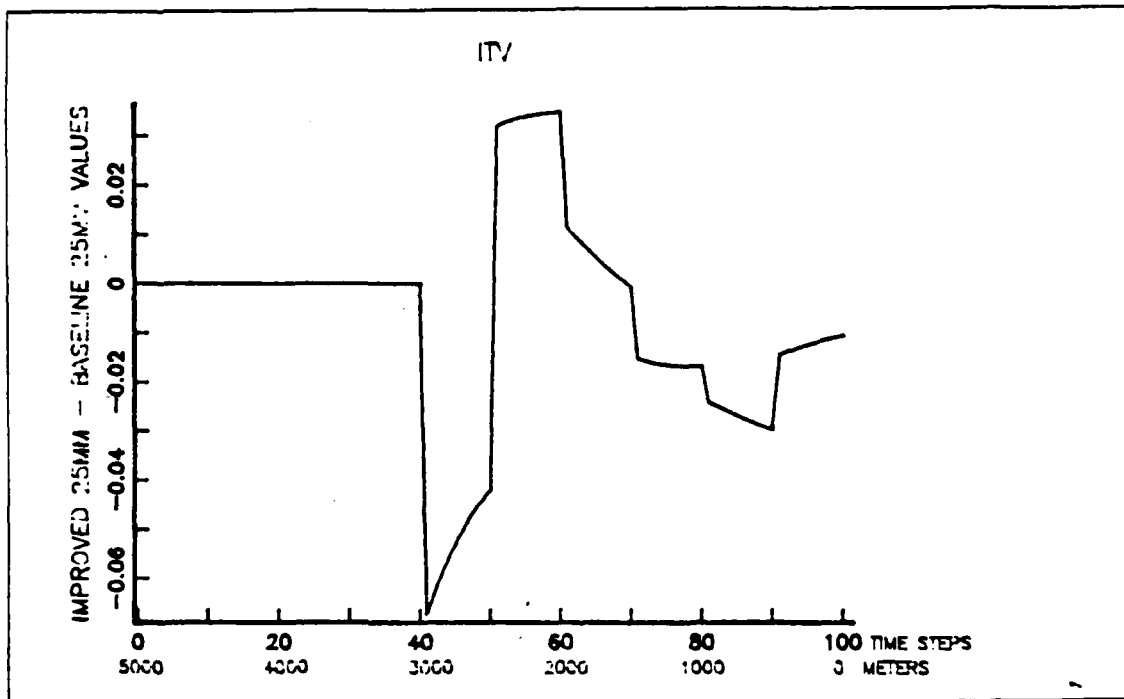
X FORCE SIZE			Y FORCE SIZE		
SYSTEM	INITIAL	FINAL	SYSTEM	INITIAL	FINAL
M1	42.0000	39.8574	T72	4.0000	0.0000
ITV	10.0000	4.3361	AT5	10.0000	0.0000
M2	6.0000	3.5038	BMP	10.0000	0.0000
155	6.0000	3.6262	122	6.0000	4.4212
203	4.0000	1.6262	152	6.0000	5.6604

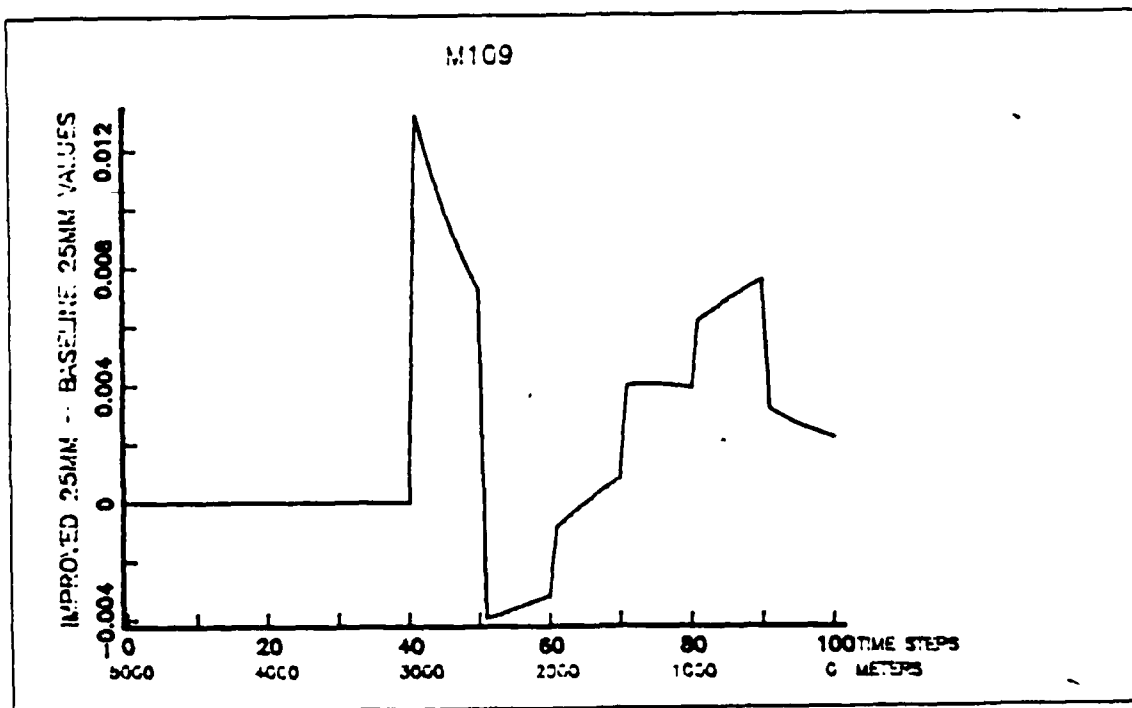
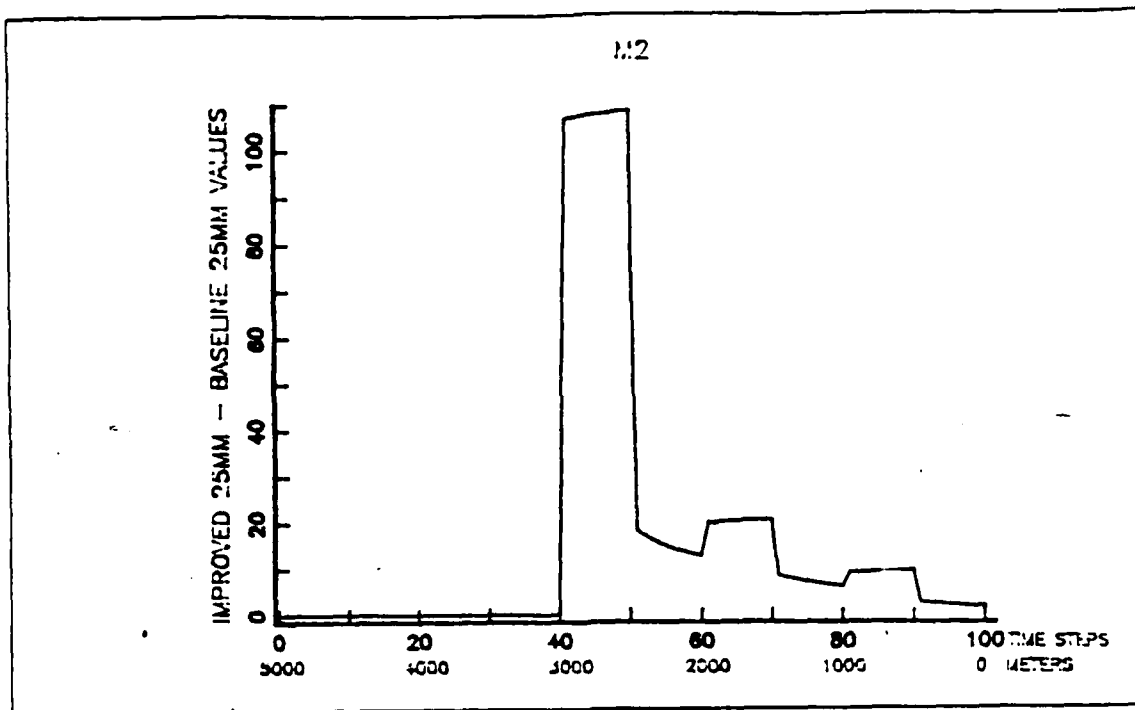
APPENDIX G

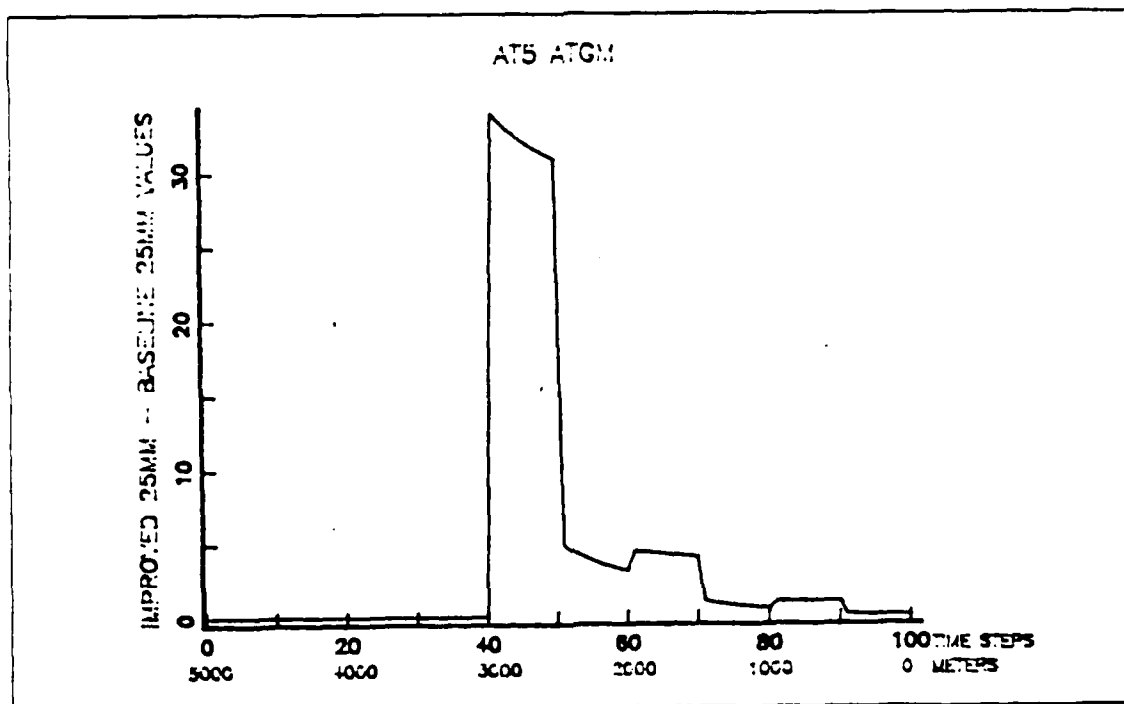
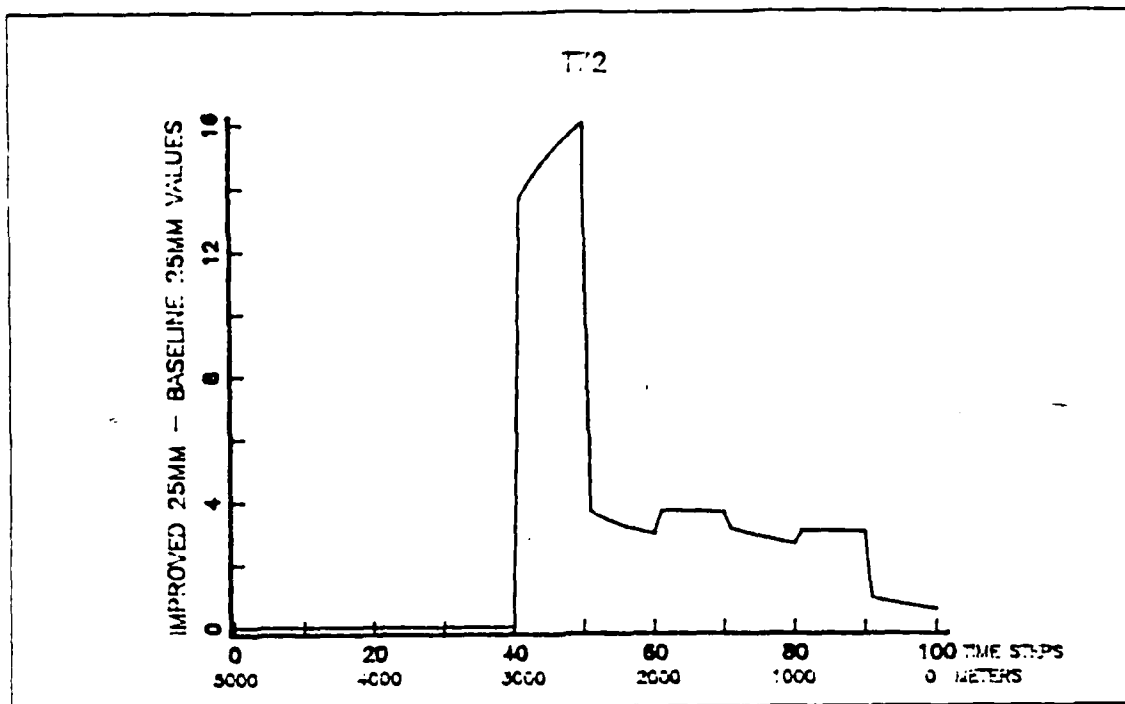
SENSITIVITY ANALYSIS PLOTS

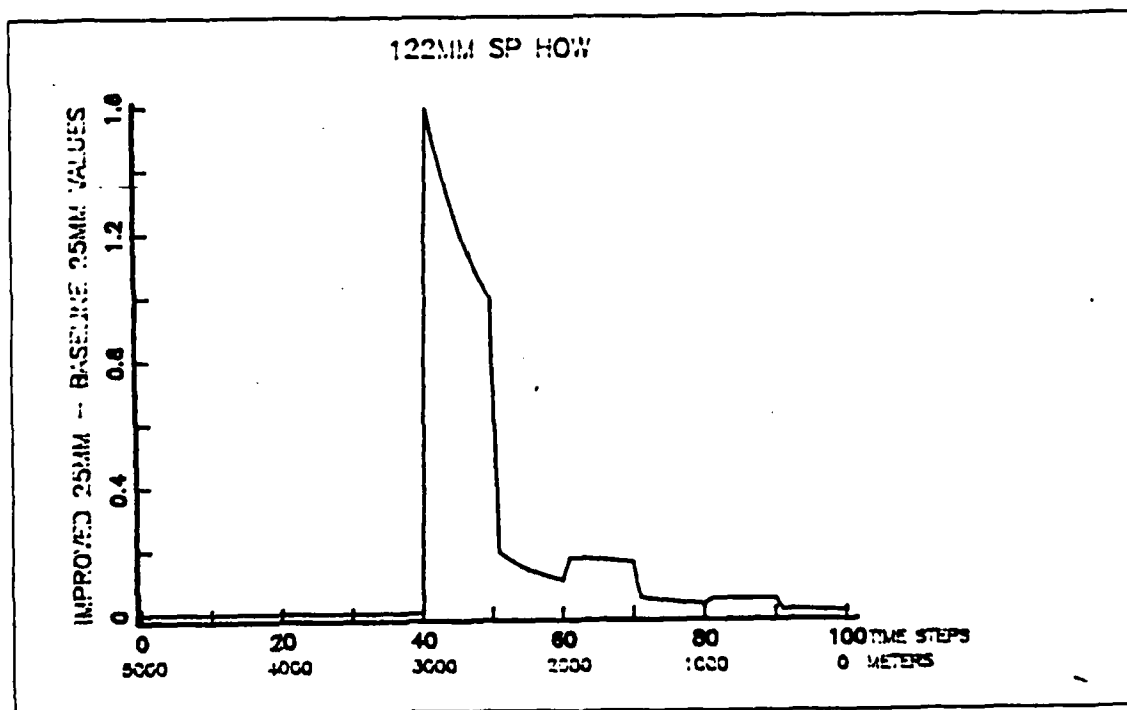
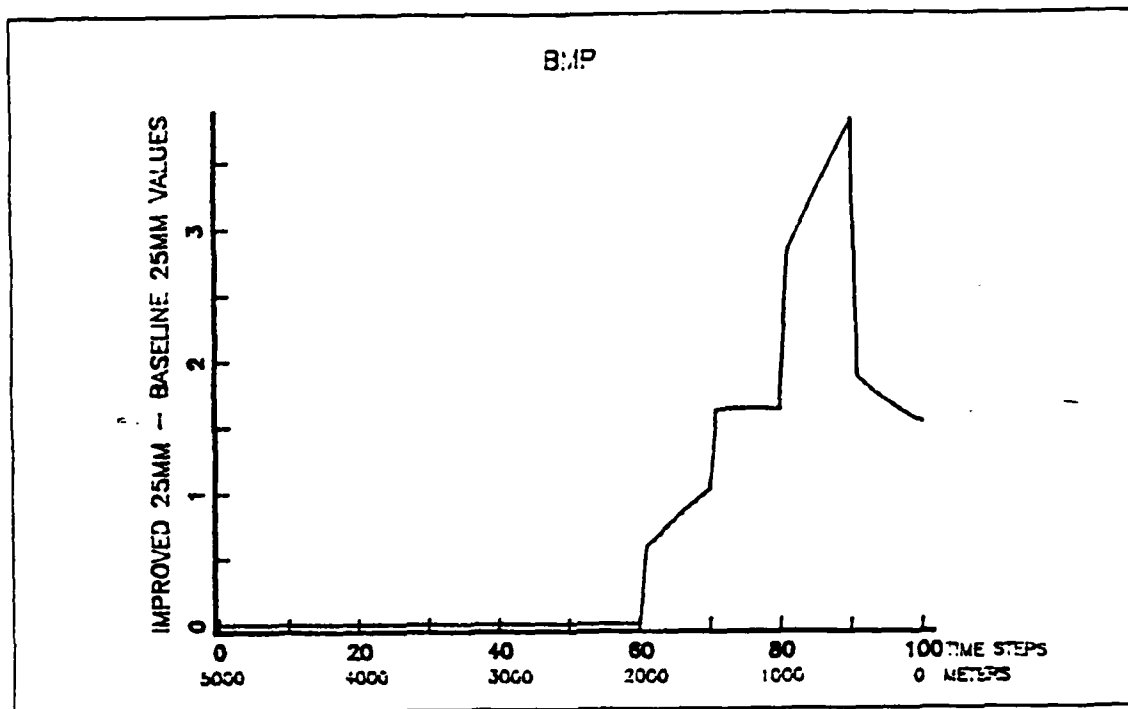
The following graphs represent the various changes identified with the introduction of the improved 25mm gun into the Lancaster and Potential-Antipotential models using the analytical FPS and Bonder equation attrition methodologies. They are provided as an appendix to illustrate some of the observations made in Chapter V. They represent only a small portion of the approaches used for the sensitivity analysis and should therefore not be considered all inclusive.

System Value Changes in Analytical FPS Simulations

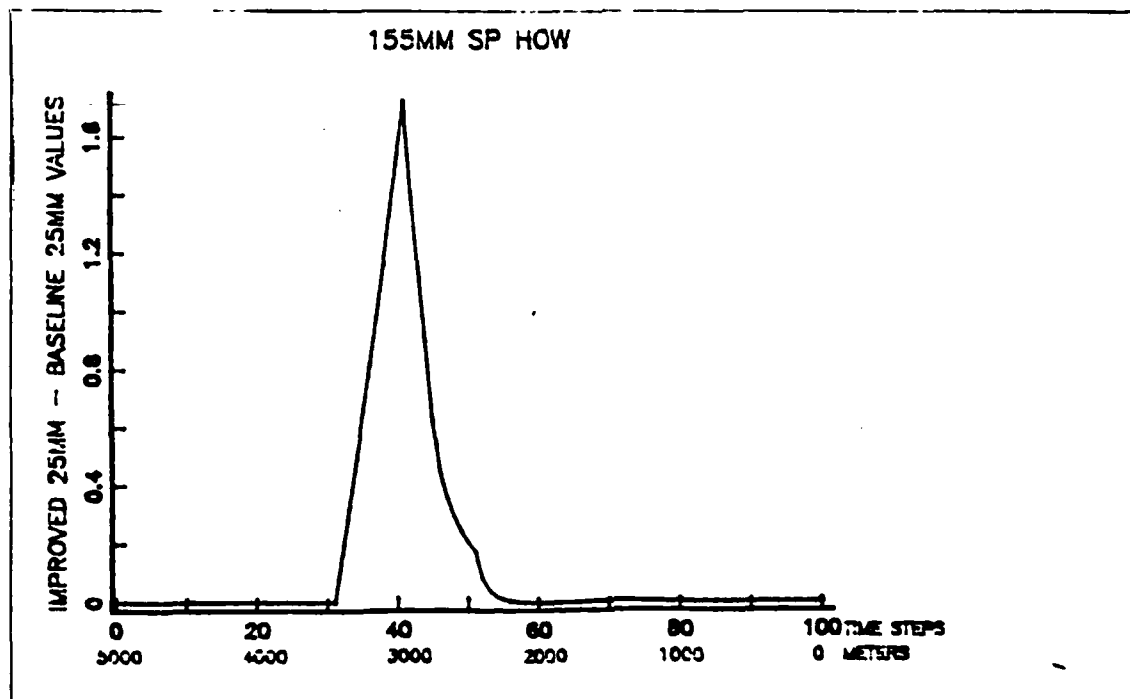
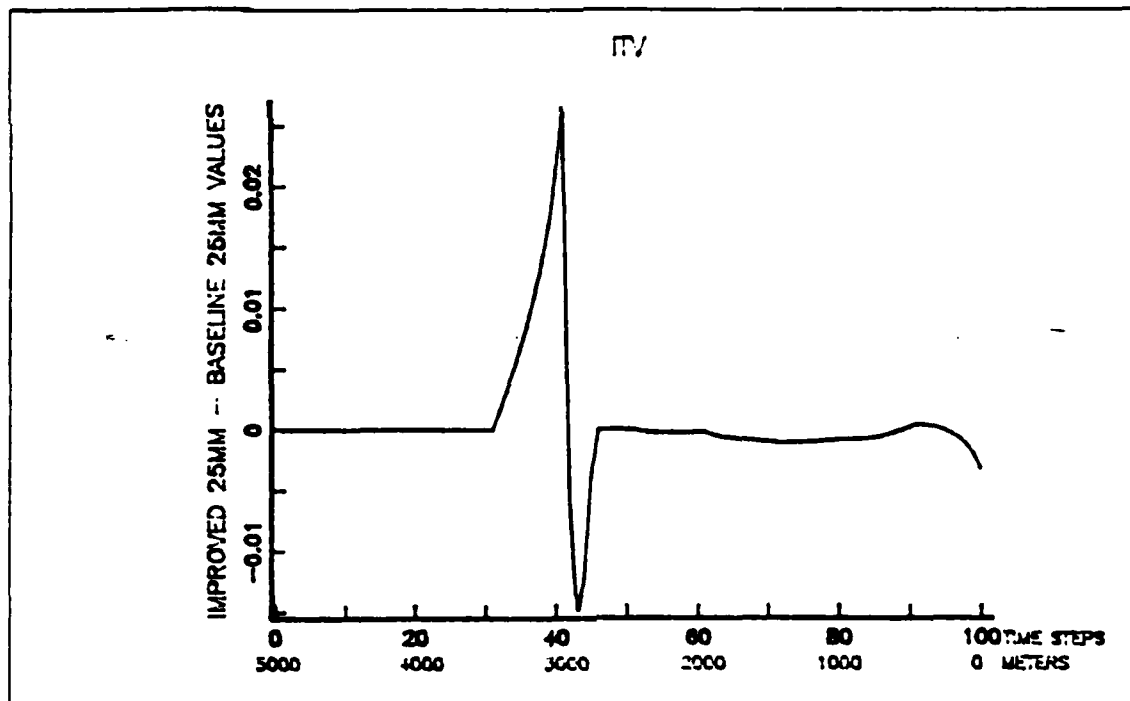


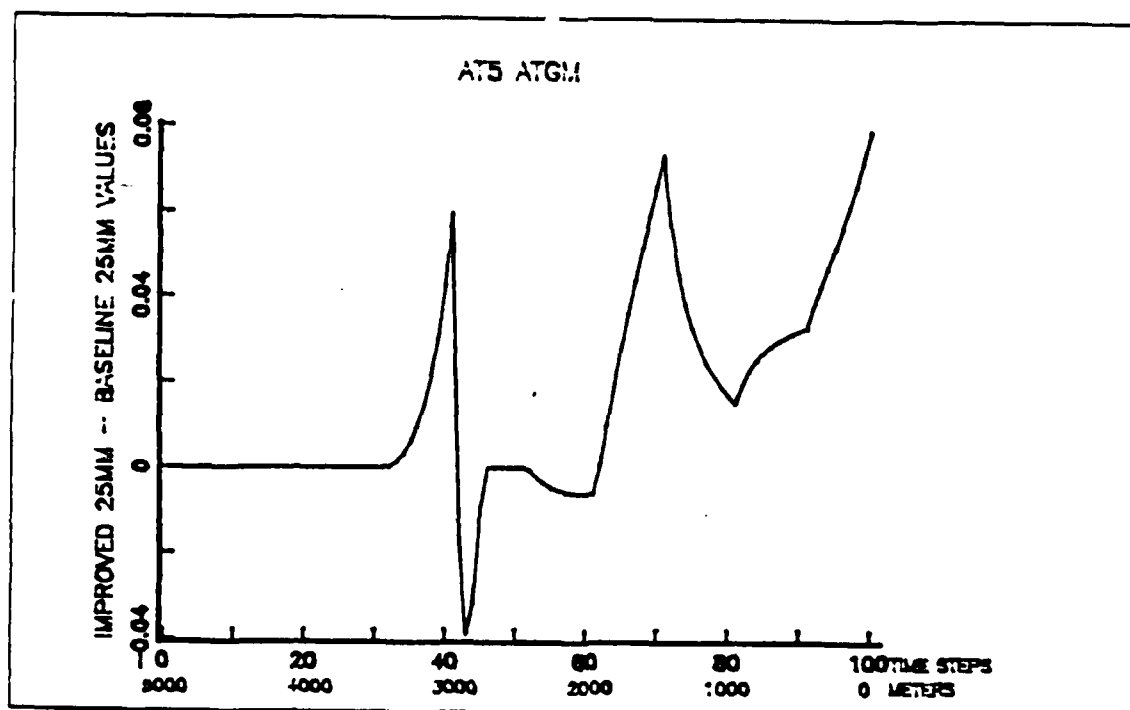
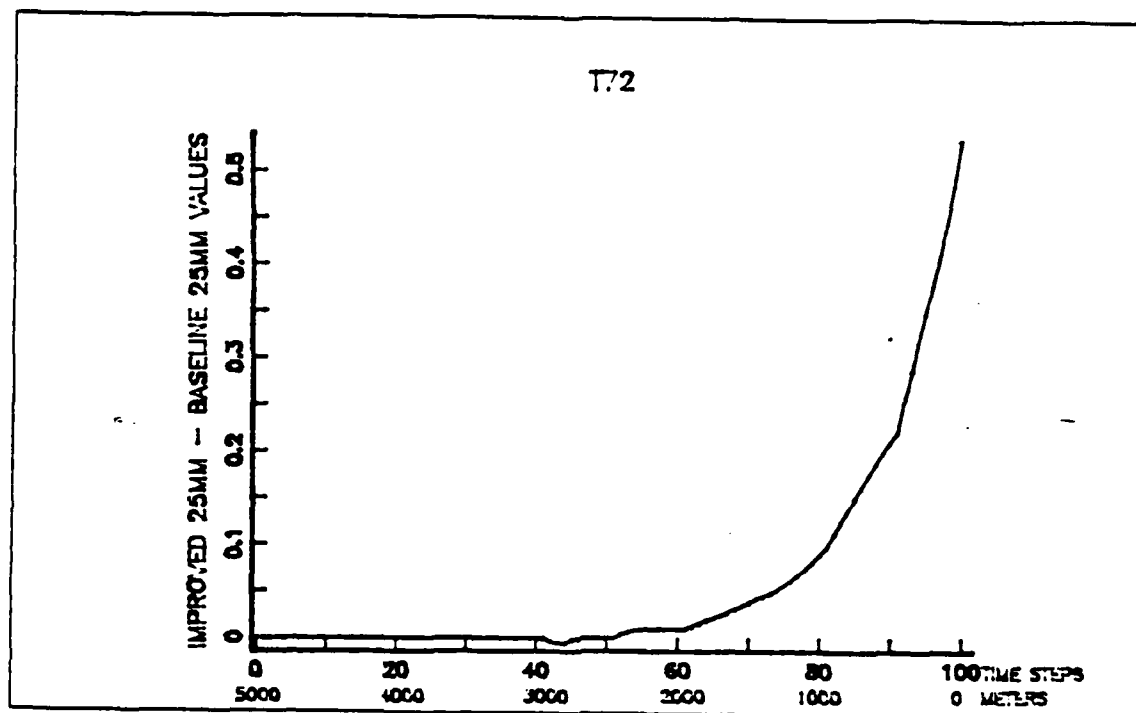


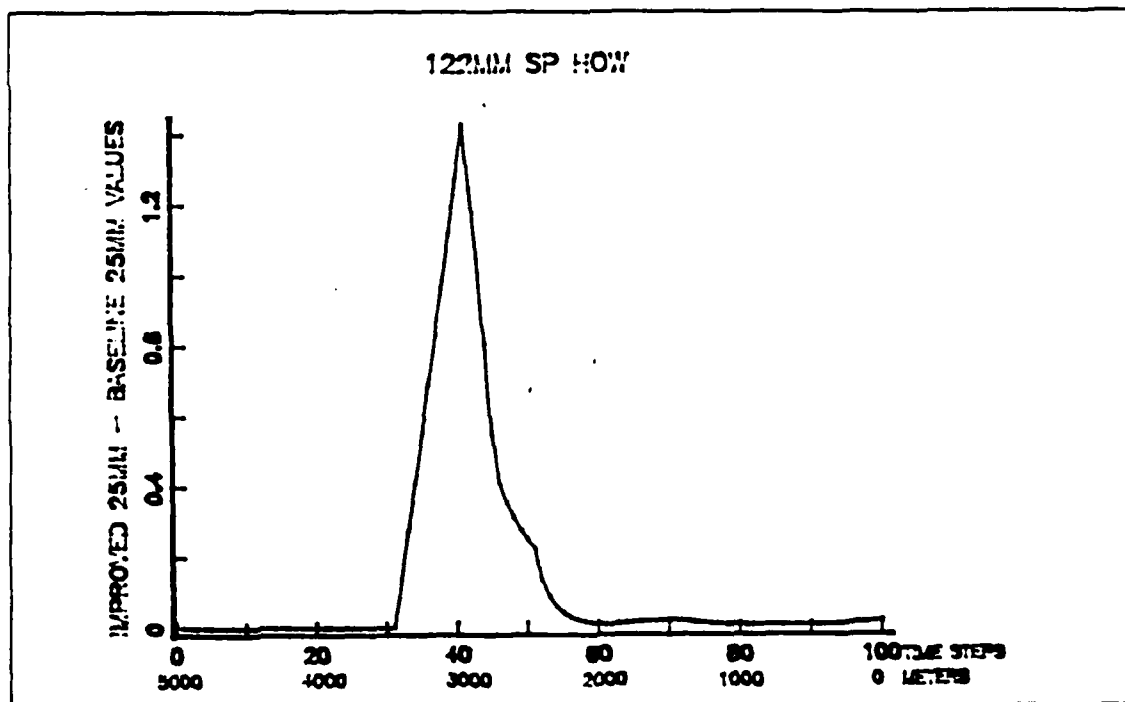
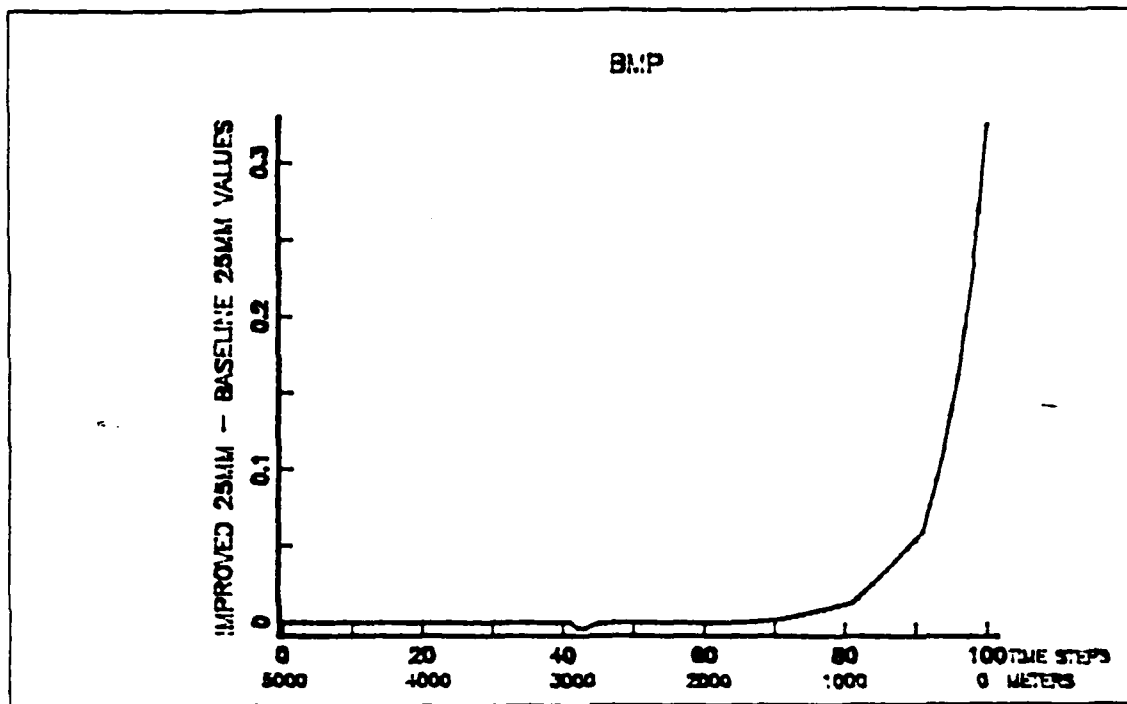


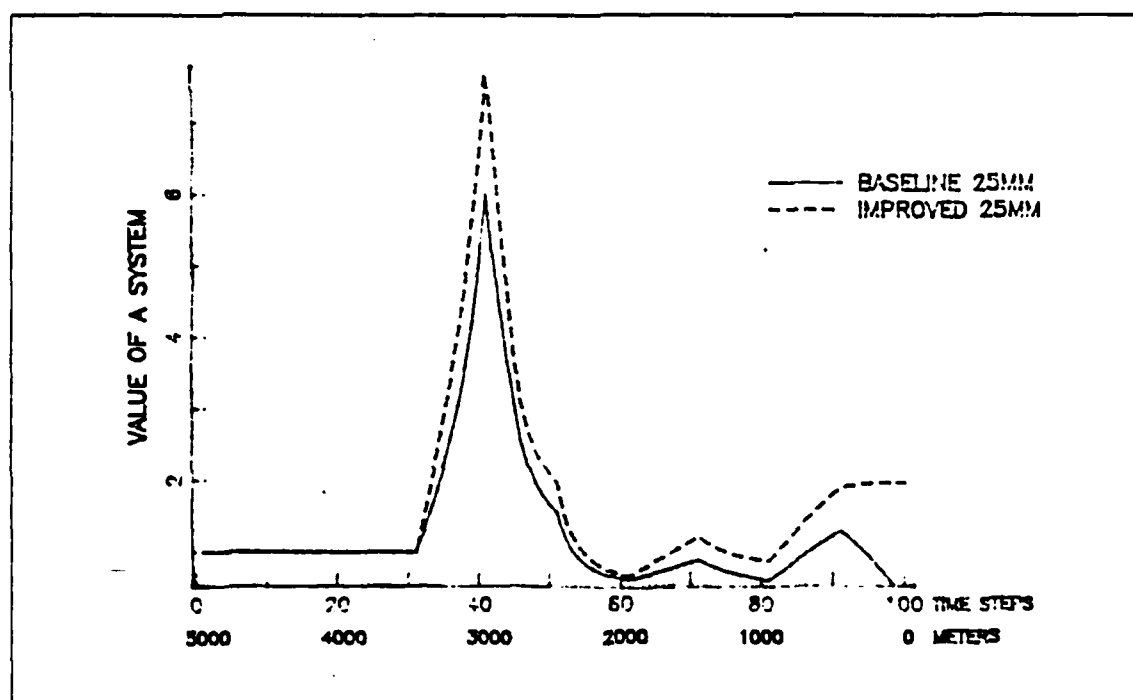


System Value Changes in Bonder P-AP Simulations









LIST OF REFERENCES

1. Parry, Samuel H., "COMSCORS Workshop", *Phalanx*, p.28-30, June 1987.
2. Hartman, James K., *Lecture Notes on Aggregated Combat Models*, Naval Postgraduate School, 1985.
3. Taylor, James G., *A Tutorial on the Determination of Single-Weapon-System-Type Kill Rates for Use in Lanchester-Type Models*, Naval Postgraduate School, August 1982.
4. Graham, Breton C., The COMANEW Model, *Proceedings of 43rd Military Operations Research Society*, p.53-62, June 1979.
5. D.M. Lester and R.F. Robinson, *Review of Index Measures of Combat Effectiveness*, Technical Paper, Office of the Under Secretary of the Army for Operations Research, 1973.
6. Bode, J.R., *Indices of Effectiveness in General Purpose Force Analysis*, BDM Corporation, BDM1W-74-070-TR, October 1974.
7. Clark, Gordon M., The Combat Analysis Model, *Proceedings 24th Military Operations Research Symposium*, p.171-180, November 1969.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3. Professor Samuel H. Parry, Code 55P Naval Postgraduate School Monterey, CA 93943-5000	2
4. Major Bard Mansager, Code 55Ma Naval Postgraduate School Monterey, CA 93943-5000	2
5. Major Kevin L. Murphy 16941 Waterbend Drive #152 Jupiter, FL 33477-5861	3

END
DATE
FILMED
JAN
1988